

Circumbinary Accretion onto Black-Hole Binaries

Dong Lai (赖东)

Cornell University & T-D Lee Institute

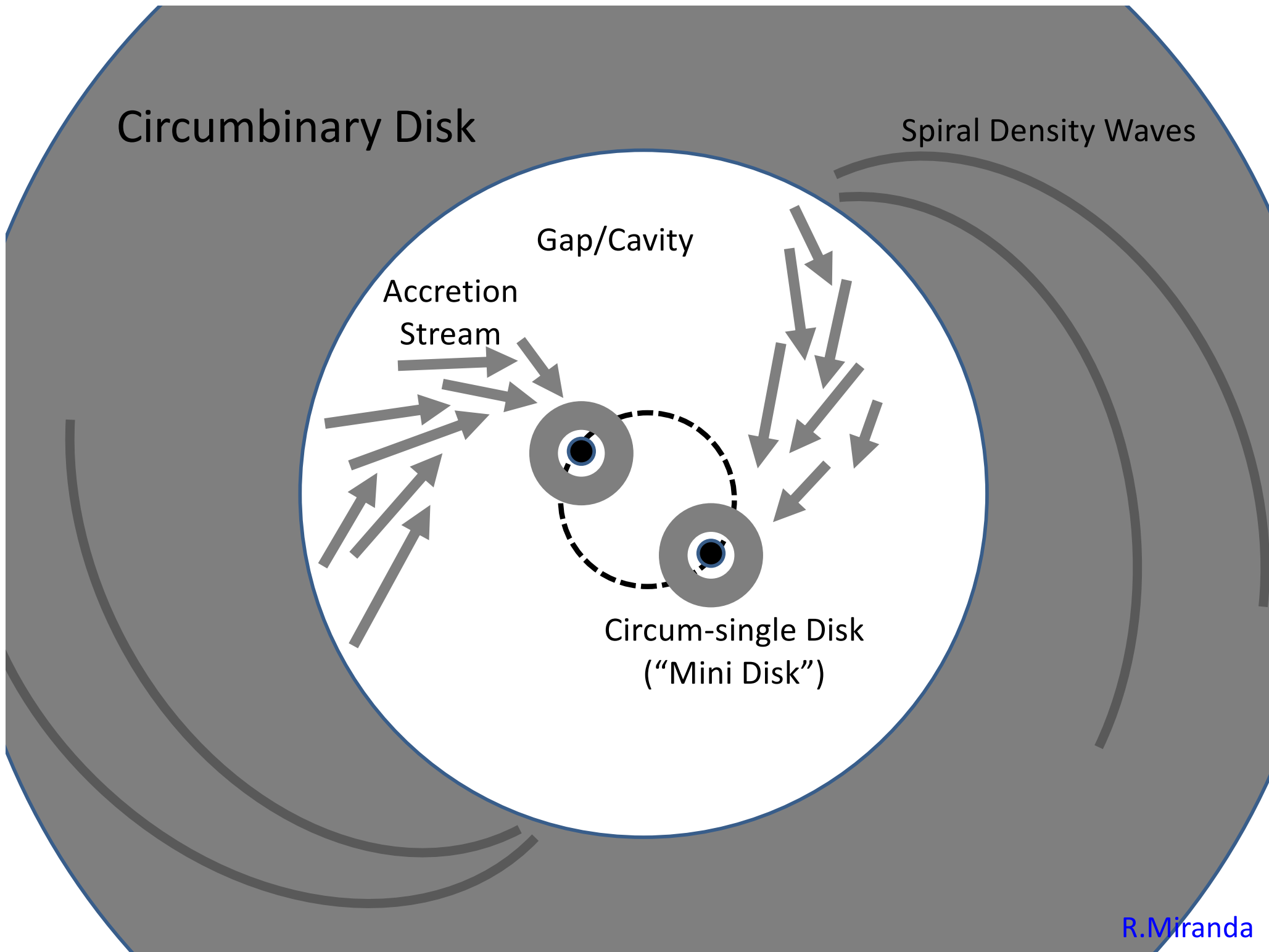
Circumbinary Disk

Spiral Density Waves

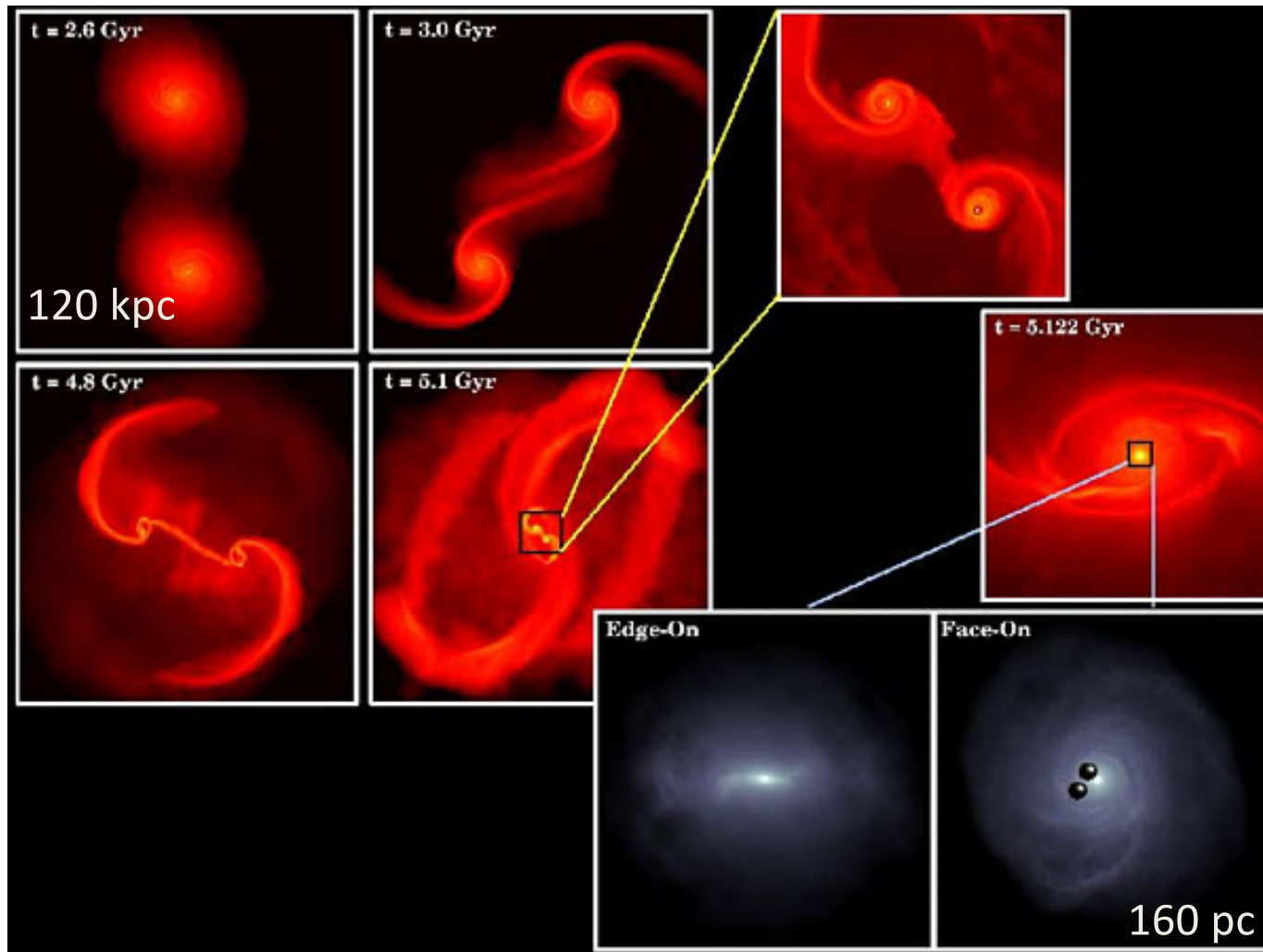
Gap/Cavity

Accretion
Stream

Circum-single Disk
("Mini Disk")



Galaxy merger → SMBH binary in gas disk/torus



Mayer et al 2007

Some key questions:

■ Accretion variability?

observational search for SMBH binaries.

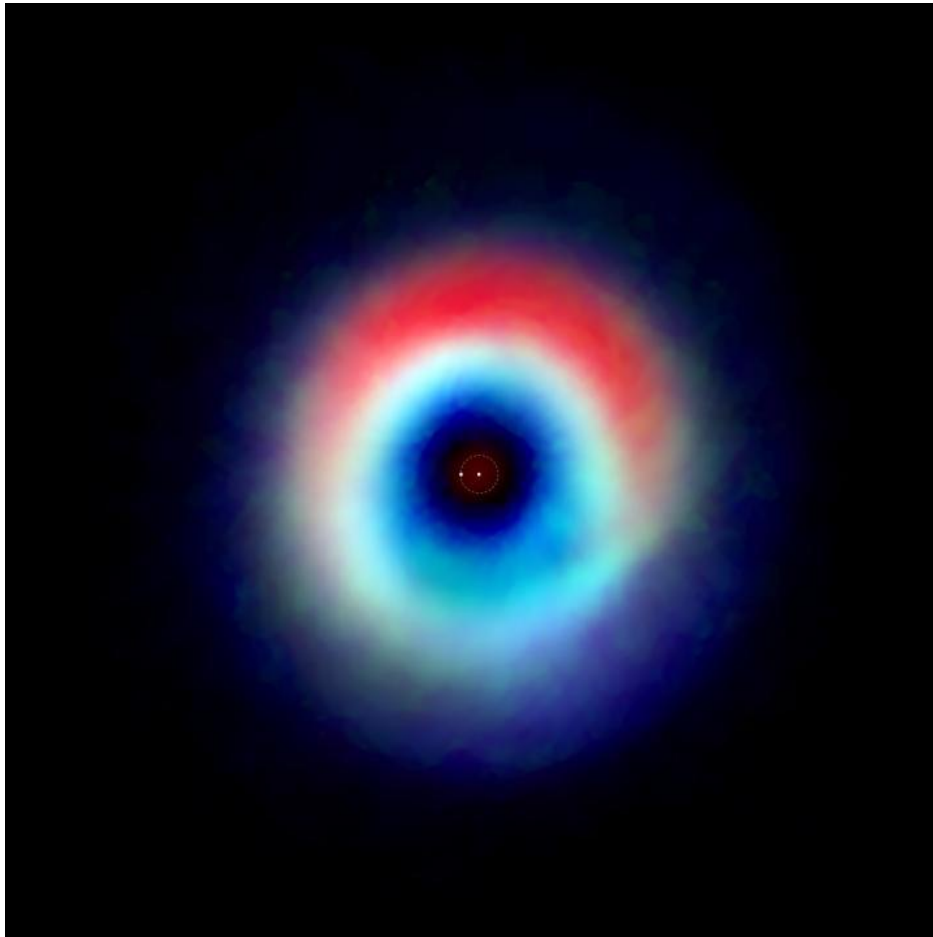
EM counterparts of low-freq. GWs (LISA, Pulsar timing arrays)

■ Long-term evolution of the binary? (Does the orbit decay?)

final pc problem (?)

Disks around proto-stellar binaries

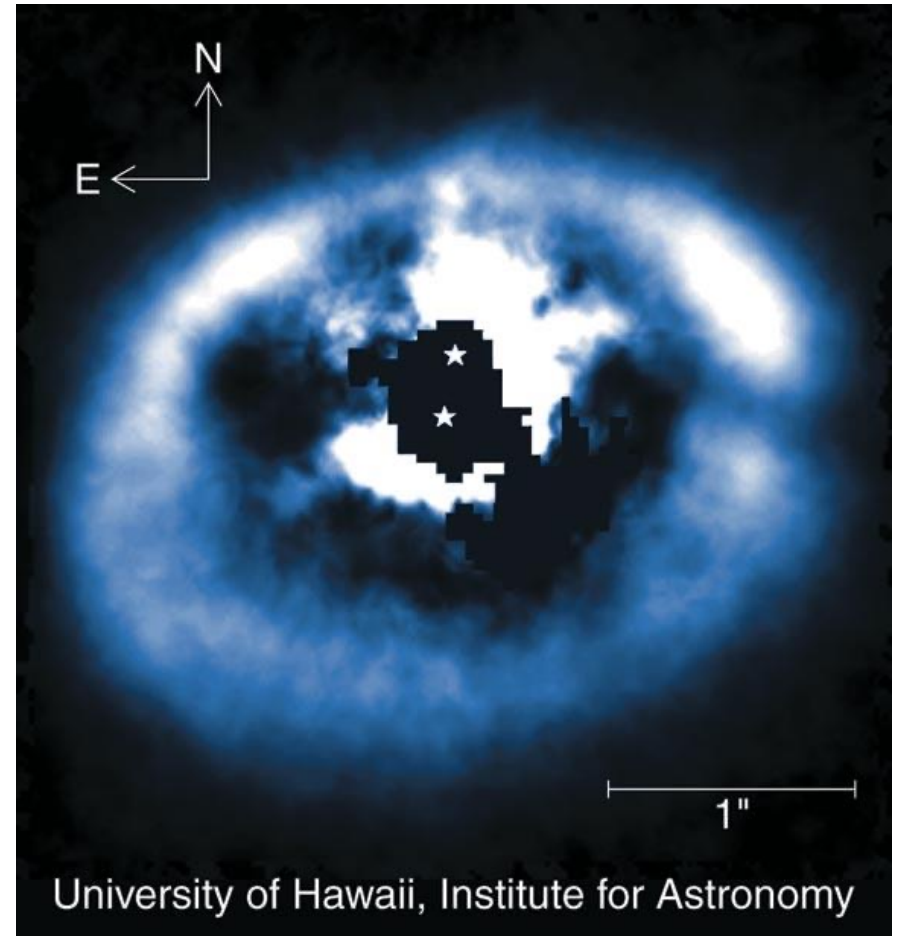
HD 142527



Outer disk : >100 AU
Gap (cavity): 10-100 AU
Inner binary: ~ 20 AU

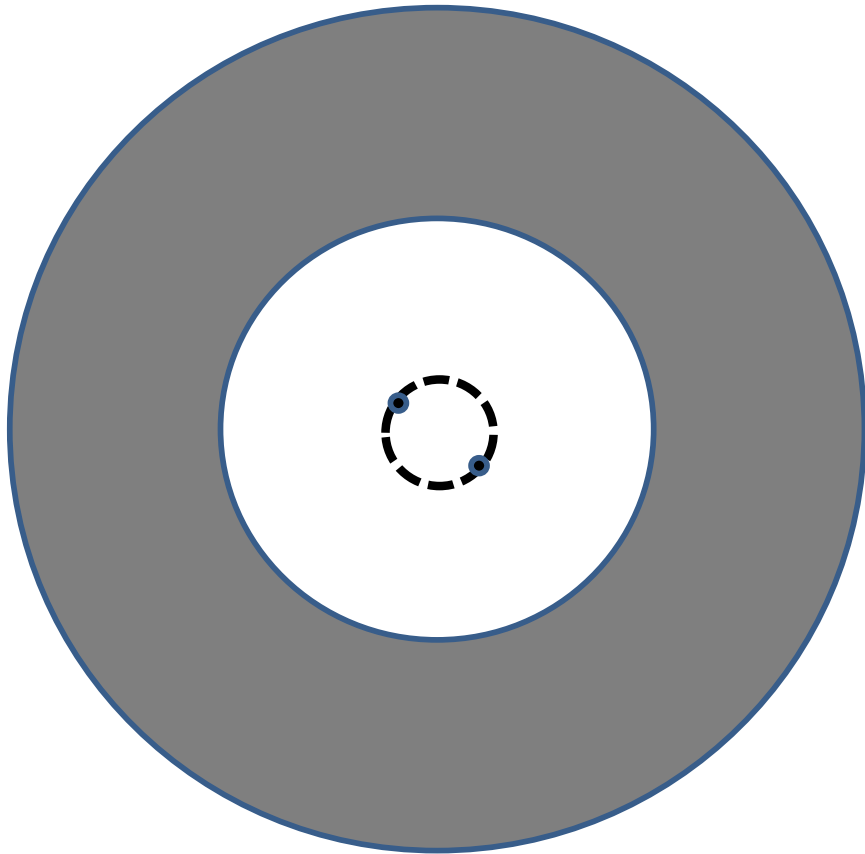
A. Isella/ALMA

GG Tau



Binary: ~ 60 AU

Disk around Binary: Gap/Cavity opening



Binary produces a gravitational potential on disk:

$$\Phi(\mathbf{r}, t) = \sum_m \Phi_{mn}(r) \cos(m\phi - m\Omega_b t)$$

Transfer angular momentum to the disk through
“Lindblad resonance”:

$$m\Omega_b - m\Omega(r) = \kappa \simeq \Omega(r)$$

→ Disk is “pushed” outward

Viscosity → disk diffuses inward

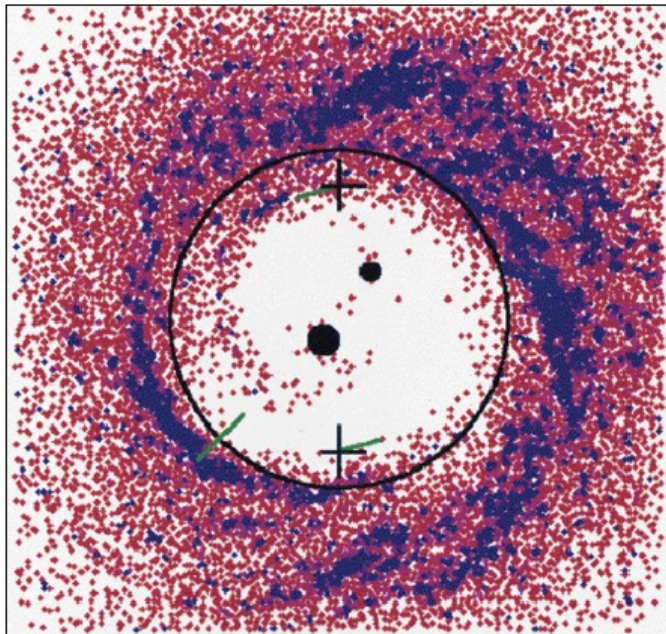
Cavity radius $\approx (2 - 3)a_b$

Simulations of Circumbinary Accretion

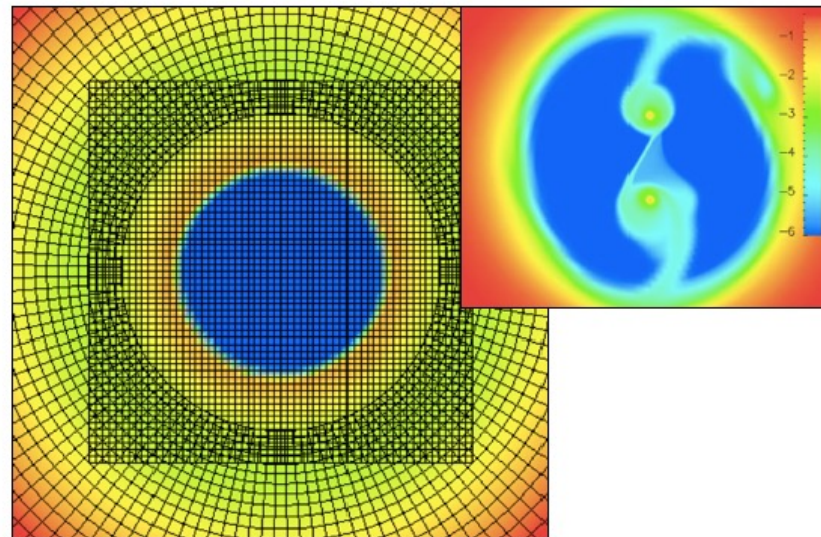
Artymowicz & Lubow 1996; Günther & Kley 02; MacFadyen & Milosavljević 08; Cuadra et al.09; Hanawa et al. 10; de Val-Borro et al. 11; Roedig et al. 12; Noble et al.12; Shi et al. 12; D’Orazio et al. 13; Pelupessy & Portegies-Zwart 13; Farris et al. 14; Shi & Krolik 15; Lines et al. 15; O’Ozario et al. 16; Ragusa et al. 16, [Munoz & Lai 2016](#); [Miranda, Munoz & Lai 2017](#); Tang et al. 17; Bowen et al.17,19; [Munoz, Miranda, Lai 2019](#); Moody, Shi & Stone 19; [Munoz, Lai et al.2020](#); Duffell et al.20; Tiede et al. 20; Heath & Nixon 20; D’Orazio & Duffell 21; Zrake et al.21; Penzlin et al.22; Siwek et al.22; [Wang et al. 2023](#); Siwek et al. 2024; Duffell et al. 2024....

Some pioneering works:

Artymowicz & Lubow (1996) – SPH



Günther & Kley (2002) – Hybrid grid



Simulations of Circumbinary Accretion

Artymowicz & Lubow 1996; Günther & Kley 02; MacFadyen & Milosavljević 08; Cuadra et al.09; Hanawa et al. 10; de Val-Borro et al. 11; Roedig et al. 12; Noble et al.12; Shi et al. 12; D’Orazio et al. 13; Pelupessy & Portegies-Zwart 13; Farris et al. 14; Shi & Krolik 15; Lines et al. 15; O’Ozario et al. 16; Ragusa et al. 16, [Munoz & Lai 2016](#); [Miranda, Munoz & Lai 2017](#); Tang et al. 17; Bowen et al.17,19; [Munoz, Miranda, Lai 2019](#); Moody, Shi & Stone 19; [Munoz, Lai et al.2020](#); Duffell et al.20; Tiede et al. 20; Heath & Nixon 20; D’Orazio & Duffell 21; Zrake et al.21; Penzlin et al.22; Siwek et al.22, [Wang et al.2023](#); Siwek et al. 2024; Duffell et al. 2024....

Many simulations excised the inner “cavity”

Some cover the whole domain: Circumbinary disk → stream → circumsingle disks:

Using finite-volume moving mesh codes:

[DISCO](#): Farris, Duffell, MacFadyen, Haiman 2014...

[AREPO](#): resolve accretion onto individual body to $0.02a_b$

(Munoz & Lai 2016; Munoz, Miranda & Lai 2019; Munoz, Lai et al 2020...)

[ATHENA++](#) (Moody, Shi & Stone 2019; Wang, Bai & Lai 2023)

Summary of Key Dynamical Results

- Short-term variabilities
- Long-term variabilities
- Angular momentum transfer and binary evolution

“Idealized” Simulations:

- Solve viscous hydrodynamic equations in 2D
- alpha viscosity, (locally) isothermal sound speed (or EOS with simple cooling)

Disk $H/r \sim 0.1$, $\alpha = 0.05 - 0.1$ (down to 0.01)

Our own works: with Diego Munoz (Harvard PhD'13->Cornell -> ... -> NAU)

Ryan Miranda (Cornell Ph.D.'17->IAS -> Data science)

Haiyang Wang (Fudan U.-> Caltech)

Xuening Bai (Tsinghua U)

Munoz & DL 2016, ApJ; Miranda, Munoz & DL 2017, MNRAS

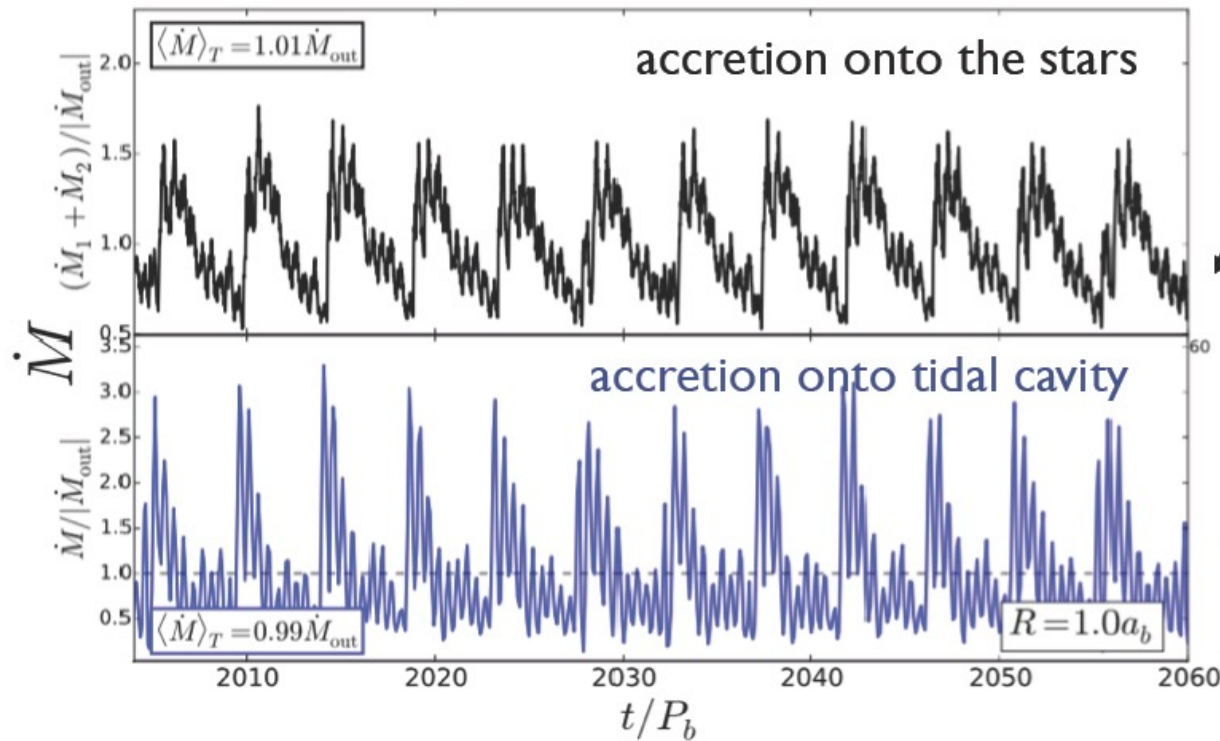
Munoz, Miranda & DL 2019; Munoz, DL et al 2020; Wang, Bai, DL et al.2023a,b

REFs: Lai & Munoz 2023 ARAA + other recent papers...

Short-term ($\sim P_b$) Accretion Variabilities

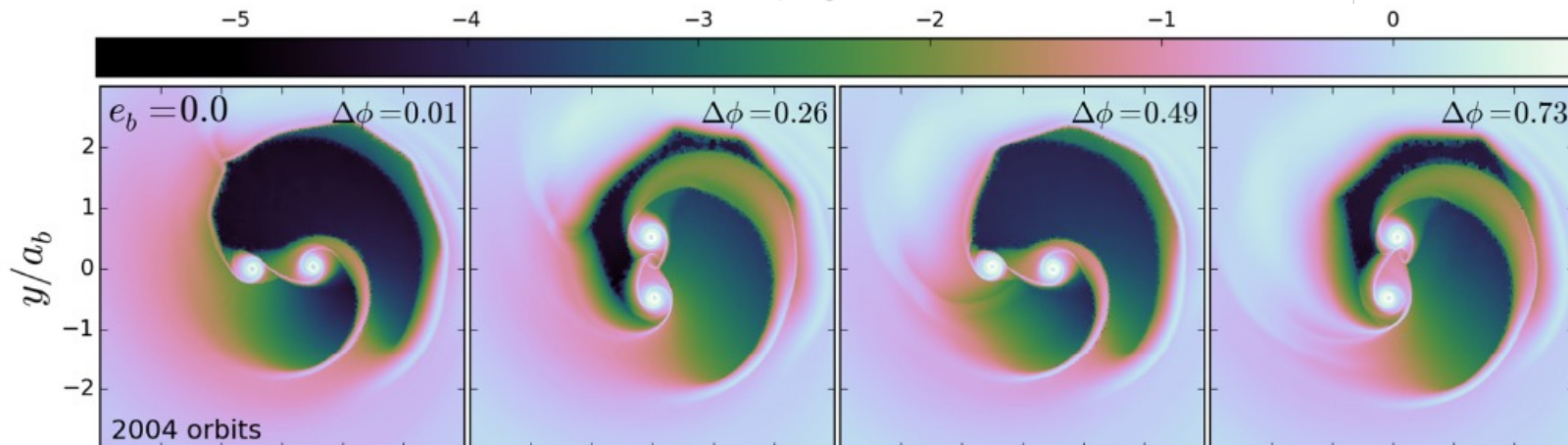
For $e_b \lesssim 0.05$: $\dot{M}(= \dot{M}_1 + \dot{M}_2)$ varies at $\sim 5P_b$ (Kepler period at $r_{in} \sim 3a_b$)

$e_b=0$



Known from
MacFadyen & Milosavljevic 08,
Shi et al.12, D'Orazio et al.13,
Farris et al.14

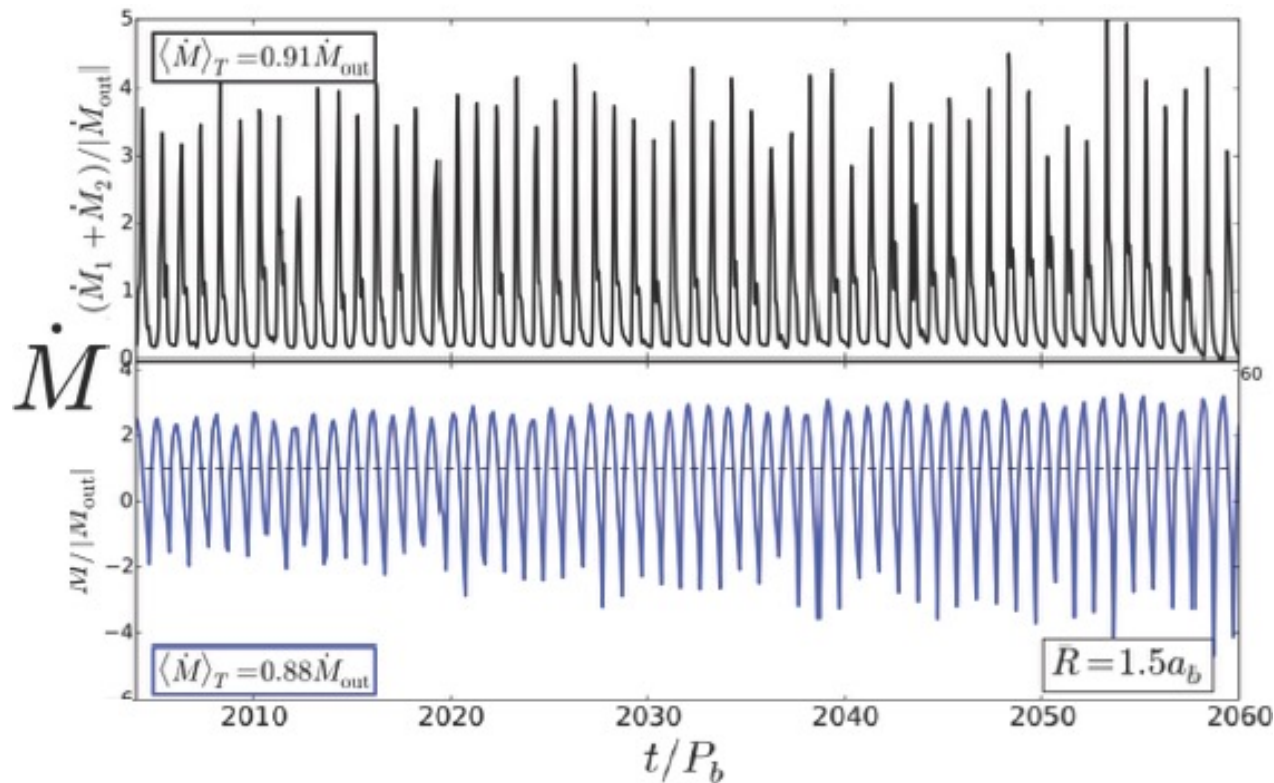
Munoz & DL 16



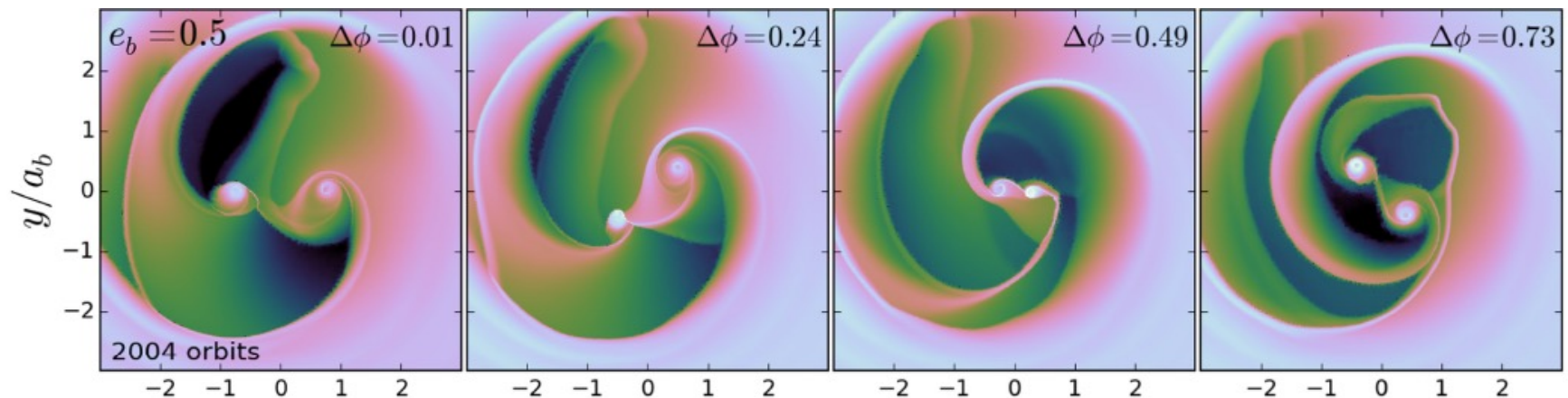
Short-term ($\sim P_b$) Accretion Variabilities

For $e_b \gtrsim 0.05$: $\dot{M} = \dot{M}_1 + \dot{M}_2$ varies at $\simeq P_b$

$e_b = 0.5$



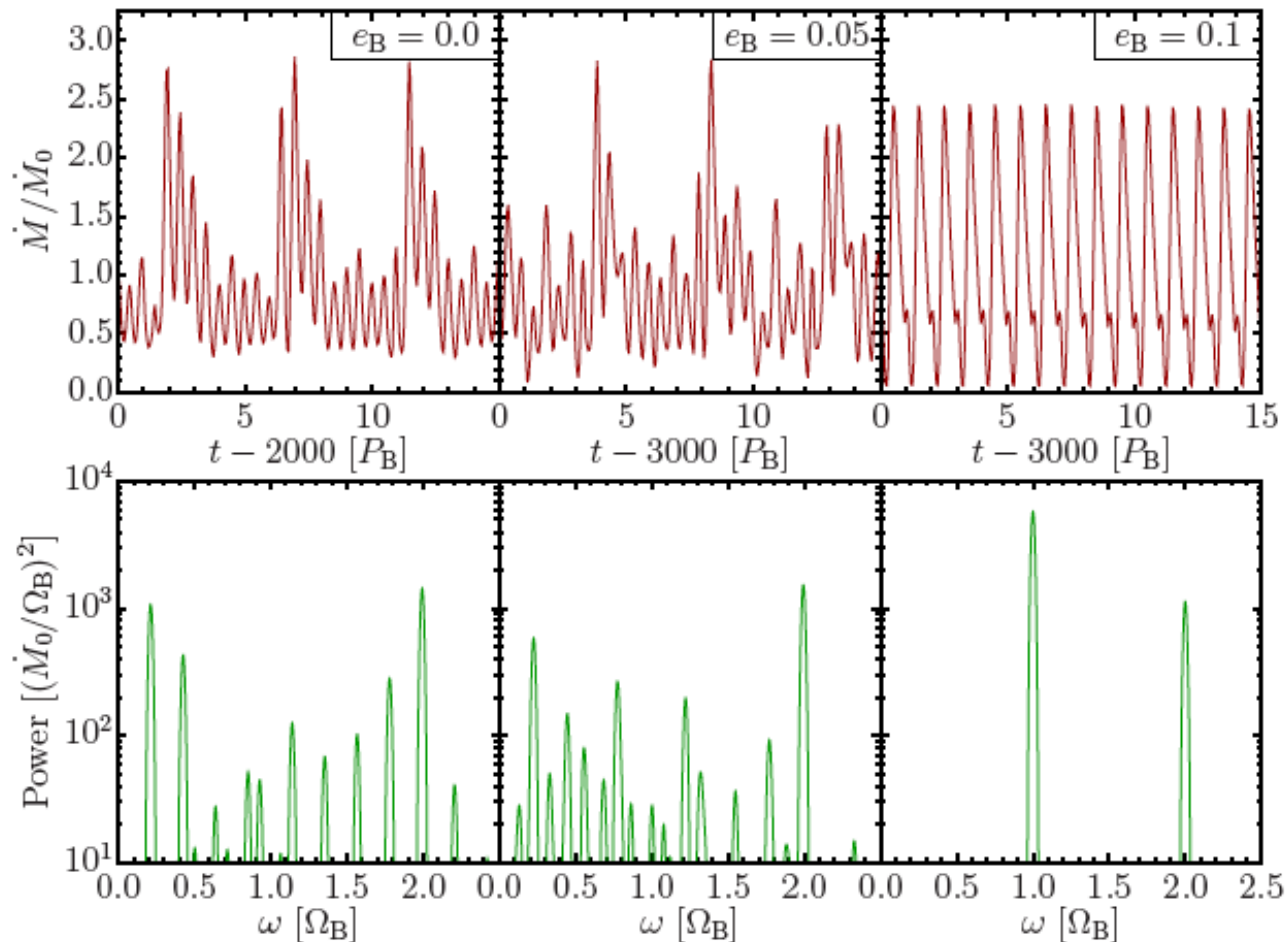
Munoz & DL 16



Short-term ($\sim P_b$) Accretion Variabilities

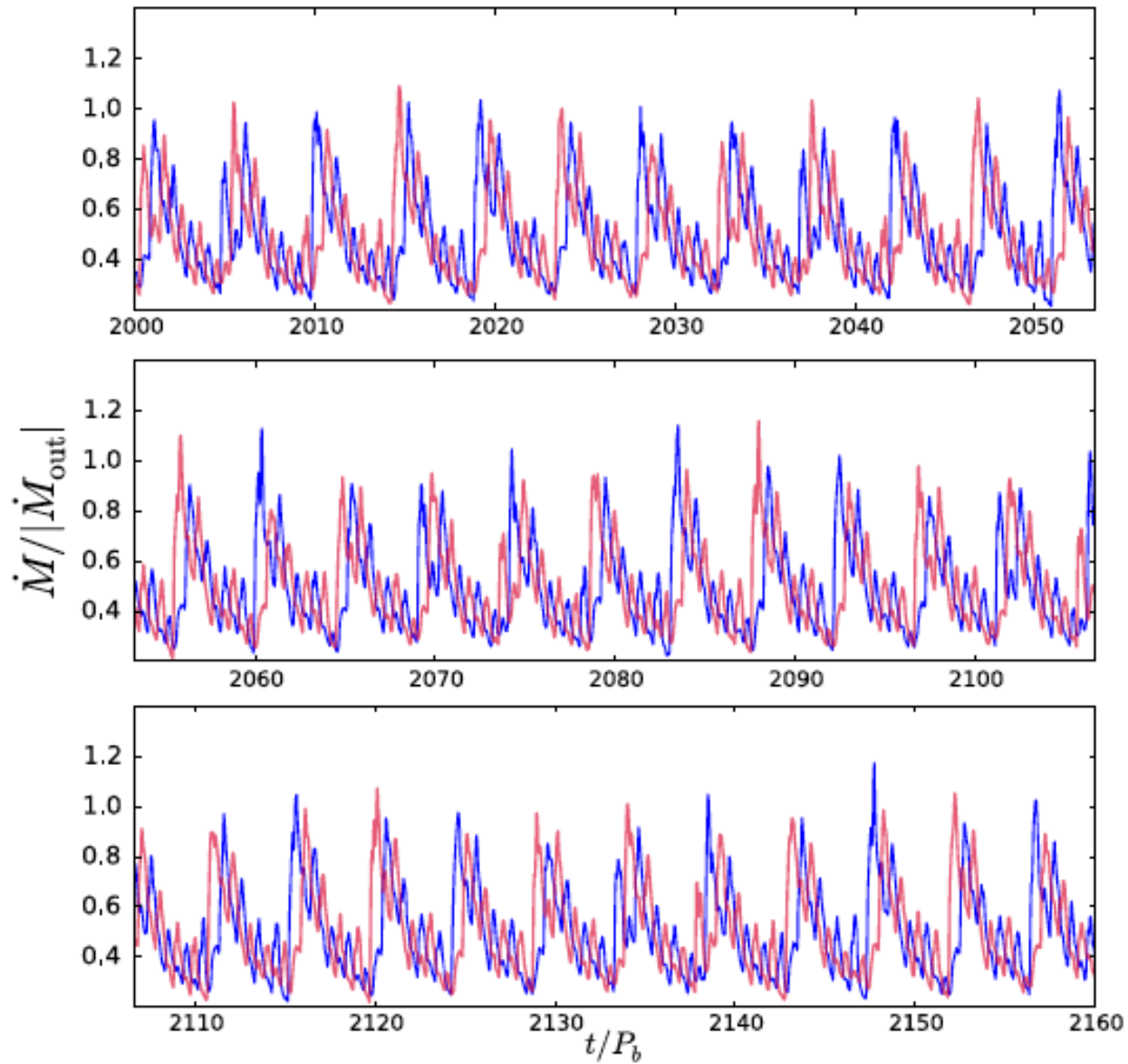
For $e_b \lesssim 0.05$: $\dot{M}(= \dot{M}_1 + \dot{M}_2)$ varies at $\sim 5P_b$

For $e_b \gtrsim 0.05$: $\dot{M} = \dot{M}_1 + \dot{M}_2$ varies at $\simeq P_b$



Power spectrum

Long-Term Variability:

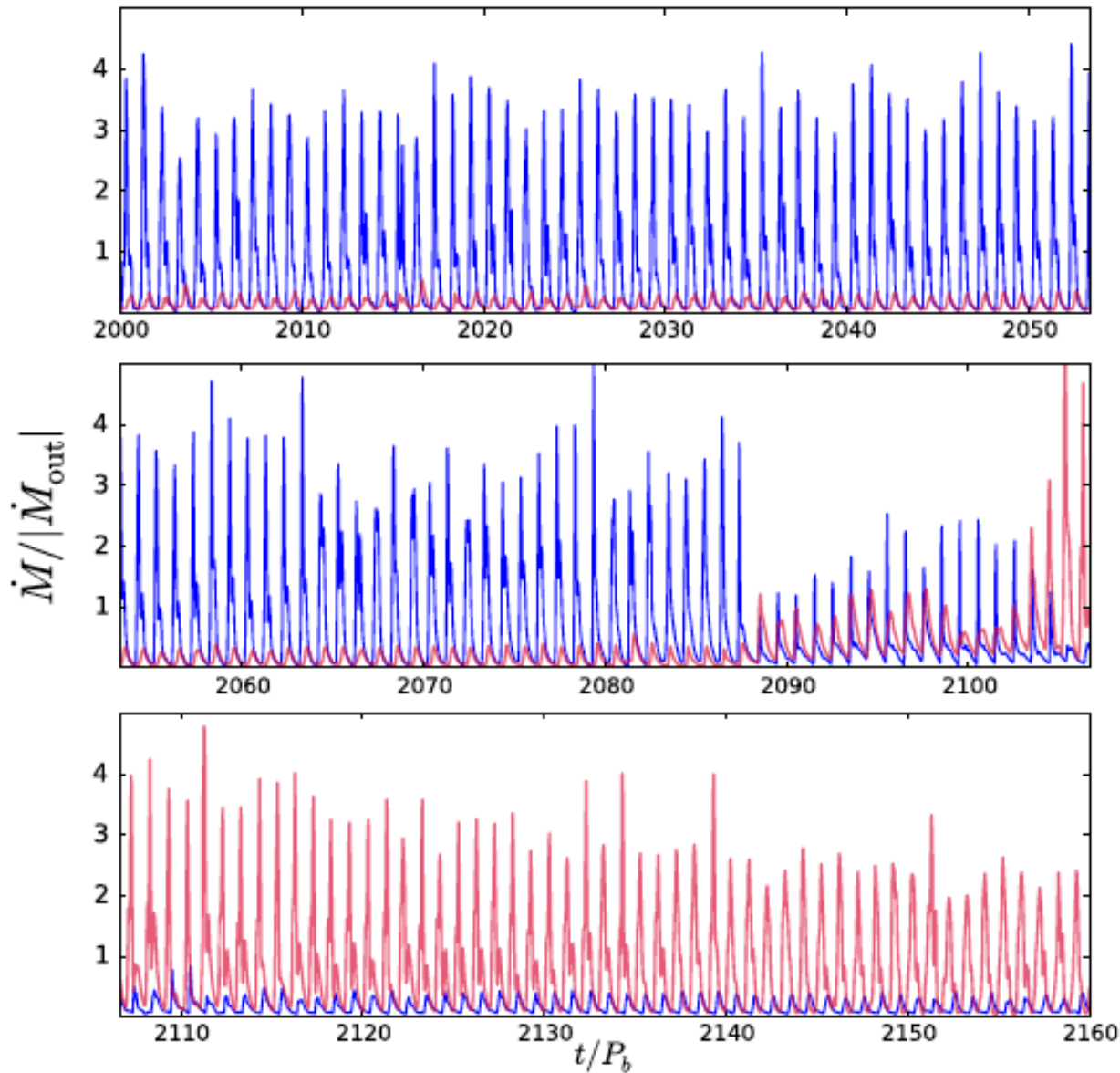


$$e_b=0$$

$$q_b=1$$

$$\dot{M}_1 \simeq \dot{M}_2$$

Long-Term Variability: Symmetry Breaking



$$e_b=0.5$$

$$q_b=1$$

Switch between

$$\dot{M}_1 \gtrsim 20\dot{M}_2$$

and

$$\dot{M}_2 \gtrsim 20\dot{M}_1$$

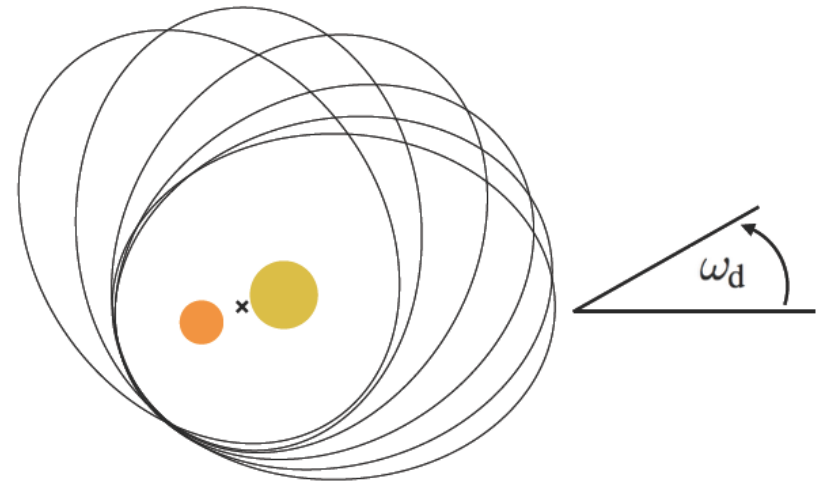
every $\sim 200 P_b$

Single AGN with binary BHs ?

Apsidal precession of eccentric disk around the binary

$$\begin{aligned}\dot{\omega}_d &\simeq \frac{3\Omega_b}{4} \frac{q_b}{(1+q_b)^2} \left(1 + \frac{3}{2}e_b^2\right) \left(\frac{a_b}{R}\right)^{7/2} \\ &\sim 0.006 \Omega_b \left(\frac{3a_b}{R}\right)^{7/2},\end{aligned}$$

Precession period 200-300 P_b



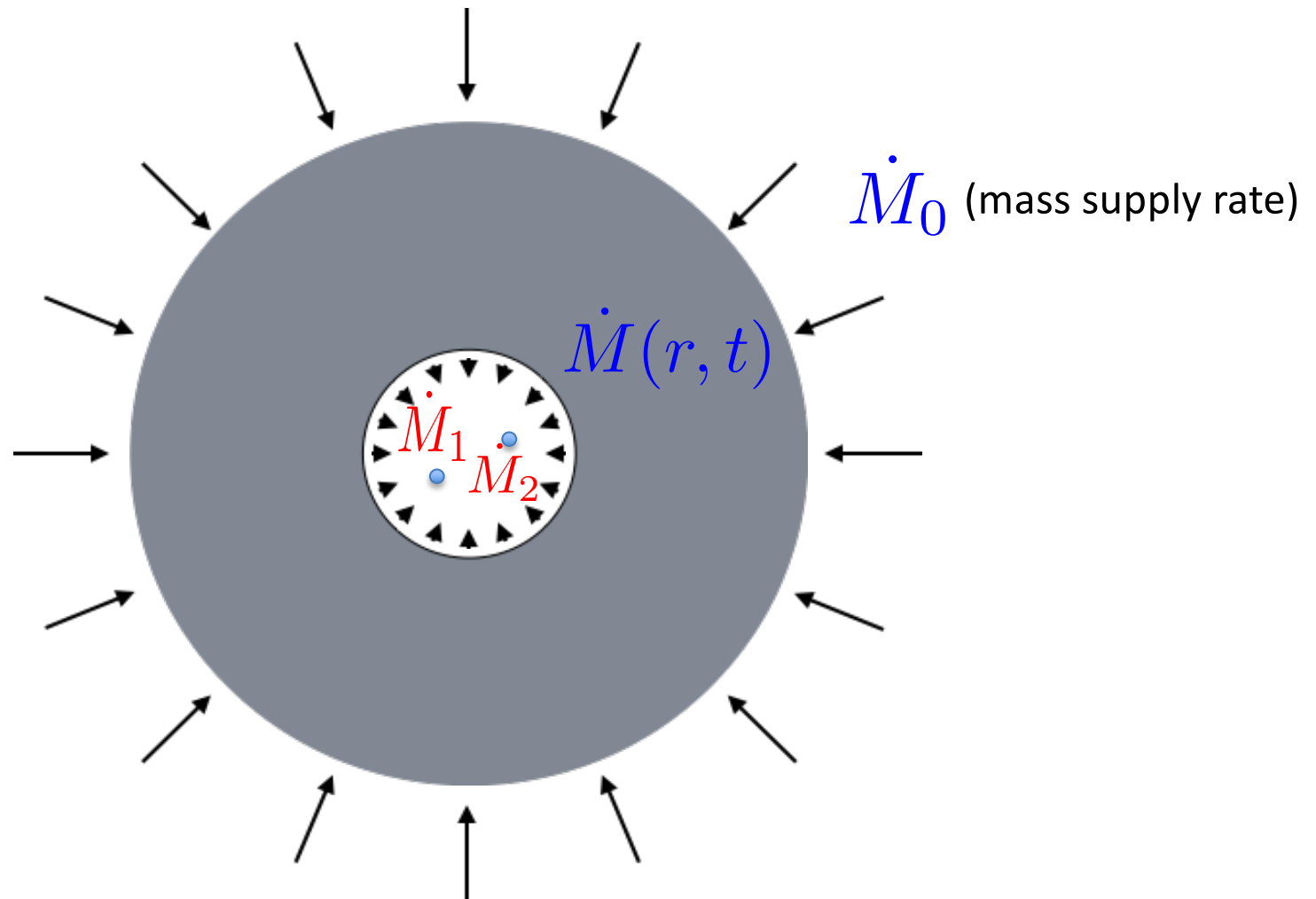
Theory of eccentric disks around binary: see Miranda, Munoz & Lai 2017
Munoz & Lithwick (2020)
Wang HY, Lai, Bai (2023)

Angular Momentum Transfer to Binary and Long-term Orbital Evolution

Many claims of orbital decays (1980s-2017):

Suppressed accretion onto binary (?), binary loses AM through outer Lindblad torque ...

First indication that the binary may expand: Miranda, Munoz & Lai 2017
(using PLUTO, excised cavity)



$\dot{M}(r, t)$, $\dot{M}_1$, $\dot{M}_2$ are highly variable

Quasi-Steady State: $\langle \dot{M}(r, t) \rangle = \langle \dot{M}_1 \rangle + \langle \dot{M}_2 \rangle = \dot{M}_0$

Two ways of computing the torque on the binary:

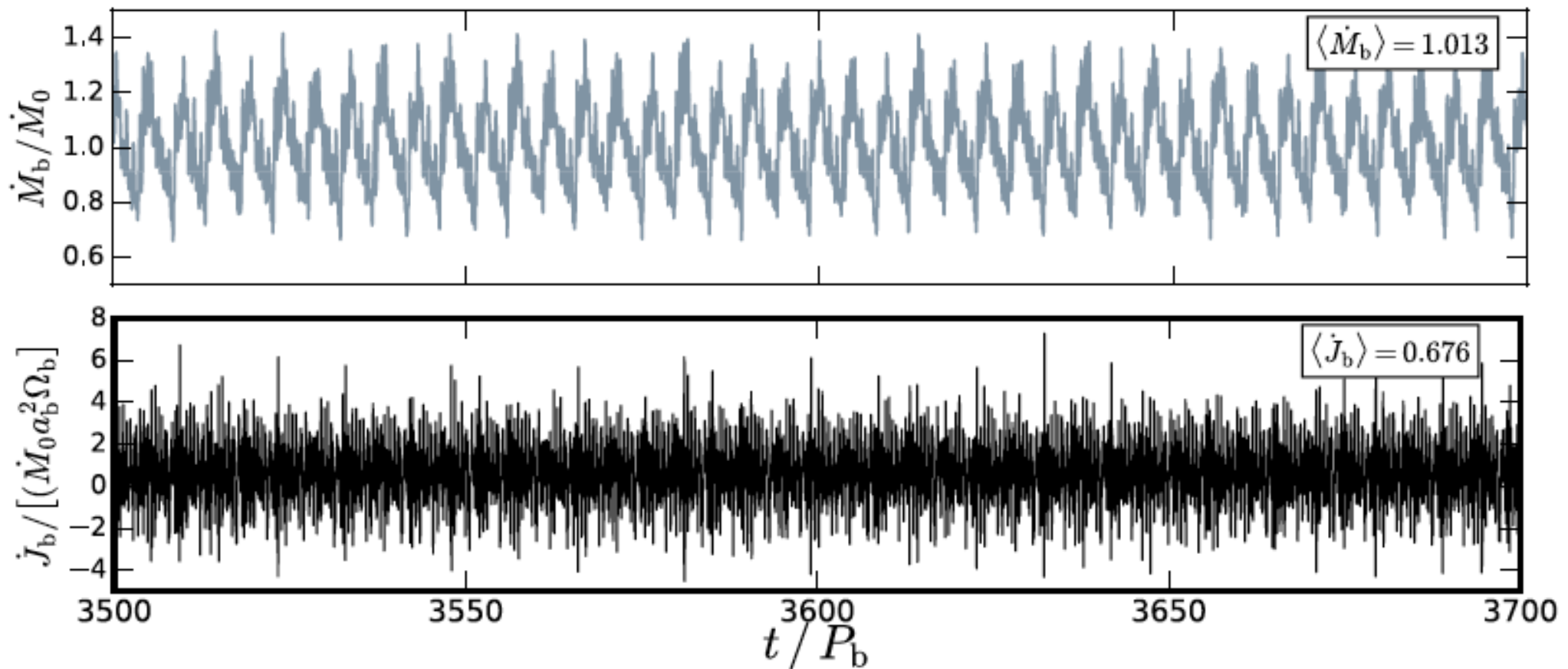
1. Direct computation:

Gravitational torque from all gas

+ Accretion torque (due momentum of accreting gas onto each star)

$$\dot{J}_b = (\dot{L}_b)_{\text{grav}} + (\dot{L}_b)_{\text{acc}} + (\dot{S}_1)_{\text{acc}} + (\dot{S}_2)_{\text{acc}}$$

Direct computation of torque on the binary



➡
$$l_0 \equiv \frac{\langle \dot{J}_b \rangle}{\langle \dot{M}_b \rangle} = 0.68 a_b^2 \Omega_b \quad e_b=0$$

Angular momentum transfer to the binary per unit accreted mass

2. Angular Momentum Current (Transfer Rate) in CBD

$$\dot{J}(r, t) = \dot{J}_{\text{adv}} - \dot{J}_{\text{visc}} - T_{\text{grav}}^{>r}$$

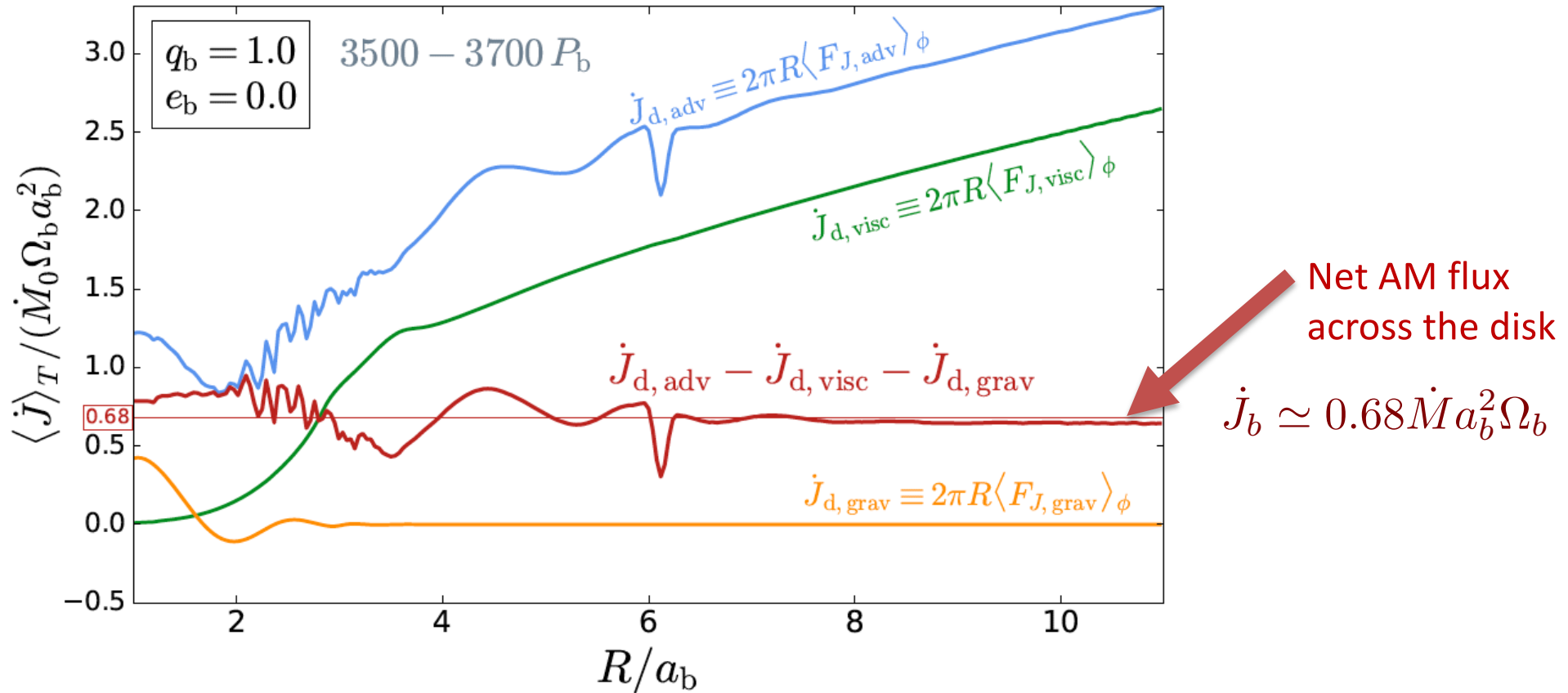
$$\dot{J}_{\text{adv}} = - \oint r^2 \Sigma u_r u_\phi d\phi$$

$$\dot{J}_{\text{visc}} = - \oint r^3 \nu \Sigma \left[\frac{\partial}{\partial r} \left(\frac{u_\phi}{r} \right) + \frac{1}{r^2} \frac{\partial u_r}{\partial \phi} \right] d\phi$$

$$T_{\text{grav}}^{>r} = \int_r^{r_{\text{out}}} \frac{dT_{\text{grav}}}{dr} dr, \quad \frac{dT_{\text{grav}}}{dr} = - \oint r \Sigma \frac{\partial \Phi}{\partial \phi} d\phi$$

2. Angular Momentum Current (Transfer Rate) in CBD

$$\dot{J}(r, t) = \dot{J}_{\text{adv}} - \dot{J}_{\text{visc}} - T_{\text{grav}}^{>r}$$



Recap: Although the accretion flow is highly dynamical, the system reaches quasi-steady state:

$$\langle \dot{M}(r, t) \rangle = \langle \dot{M}_1 \rangle + \langle \dot{M}_2 \rangle = \dot{M}_0$$

$$\langle \dot{J}_b \rangle \simeq \langle \dot{J}_{\text{disk}}(r, t) \rangle = \text{const}$$

Angular momentum transferred to the binary per unit accreted mass:

$$l_0 \equiv \frac{\langle \dot{J}_b \rangle}{\langle \dot{M}_b \rangle} = 0.68 a_b^2 \Omega_b \quad (\text{for } \alpha=0.05-0.1, H/r=0.05-0.1)$$

Munoz, Miranda & DL 2019

Confirmed by Moody, Shi & Stone 2019 (ATHENA++)
Duffell et al. (2020,2024),

Implication of $\dot{J}_B > 0$:

For $q = 1$, $e_B = 0$ binary:

$$\dot{J}_B = \dot{M}_B l_0 \quad l_0 \simeq 0.68 l_B \quad \text{where } l_B = a_B^2 \Omega_B$$

$$\rightarrow \frac{\dot{a}_B}{a_B} = 8 \left(\frac{l_0}{l_B} - \frac{3}{8} \right) \frac{\dot{M}_B}{M_B}$$

Binaries can expand due to circumbinary accretion !

$$\text{For } e_B=0: \quad \frac{\dot{a}_B}{a_B} \simeq 2.68 \frac{\dot{M}_B}{M_B}$$

Eccentric Binaries

To obtain $\dot{a}_b$ and $\dot{e}_b$, we need $\dot{J}_b$ and $\dot{E}_b$

$$\mathcal{E}_b \equiv \frac{1}{2} \dot{\mathbf{r}}_b^2 - \frac{GM_b}{r_b} \quad \text{where } \mathbf{r}_b = \mathbf{r}_1 - \mathbf{r}_2, \quad M_b = M_1 + M_2$$

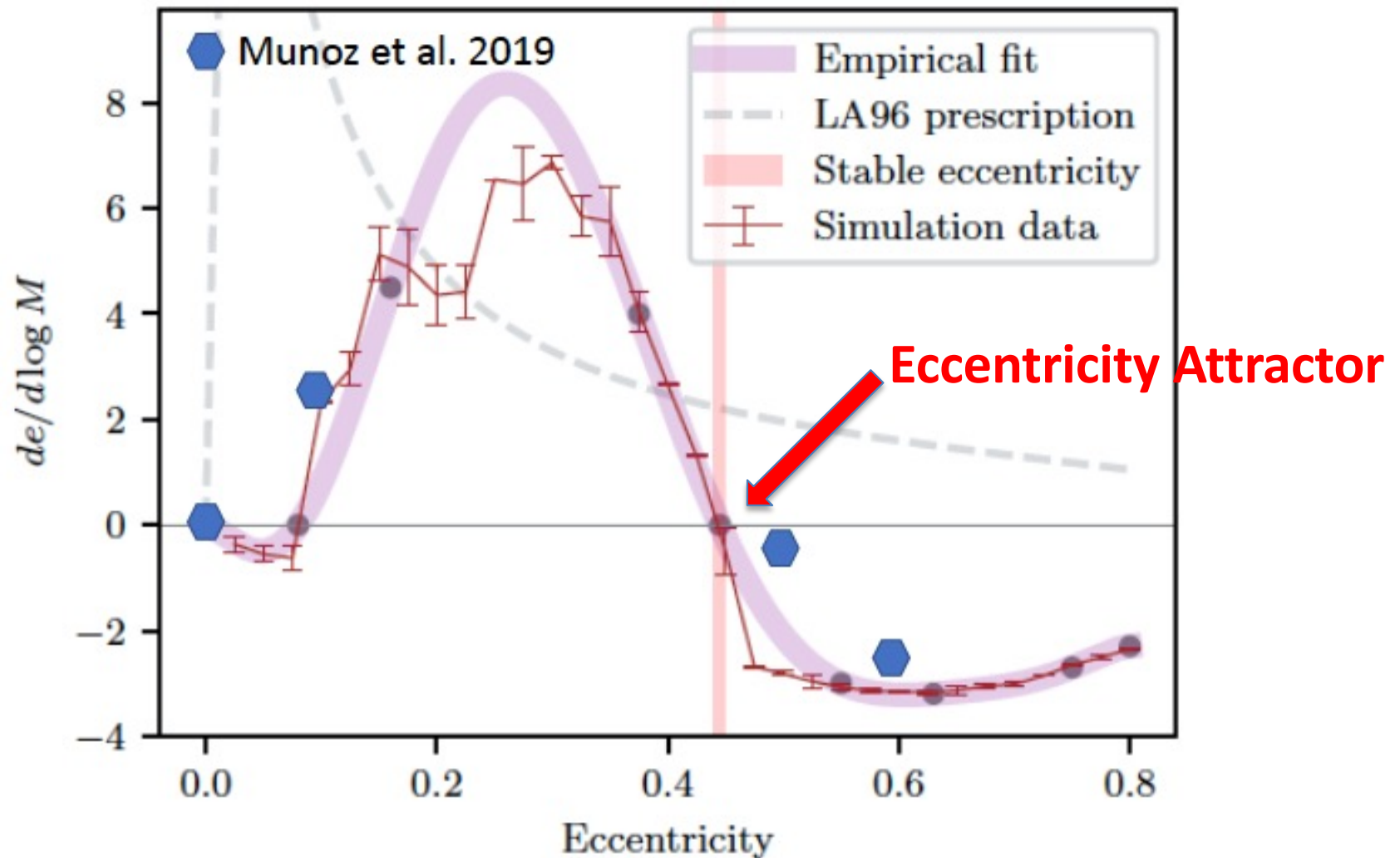
➡ $\frac{d\mathcal{E}_b}{dt} = -\frac{G\dot{M}_b}{r_b} + \dot{\mathbf{r}}_b \cdot (\mathbf{f}_1 - \mathbf{f}_2)$

$$\mathbf{f}_1 = (\text{force/mass on } M_1) = \mathbf{f}_{1,\text{gravity}} + \mathbf{f}_{2,\text{accretion}}$$

Munoz et al. 2019

e_b	$\dot{J}_b [\dot{M}_b a_b^2 \Omega_b]$	$\dot{a}_b/a_b [\dot{M}_b/M_b]$	$\dot{e}_b [\dot{M}_b/M_b]$
0	0.68	2.2	0.0
0.1	0.43	0.75	2.4
0.5	0.78	0.95	-0.20
0.6	0.81	0.47	-2.34

Eccentric Binaries



Zrake et al. 2021

See also D'Orazio & Duffell 2021

Siwek et al. 2023, 2024

Unequal-mass binaries

$$q = M_2/M_1 < 1$$

$$e_b = 0$$

[Munoz, Lai, Kratter, Miranda 2020](#)

See also Duffell+2020,2024;

Siwek et al 2024

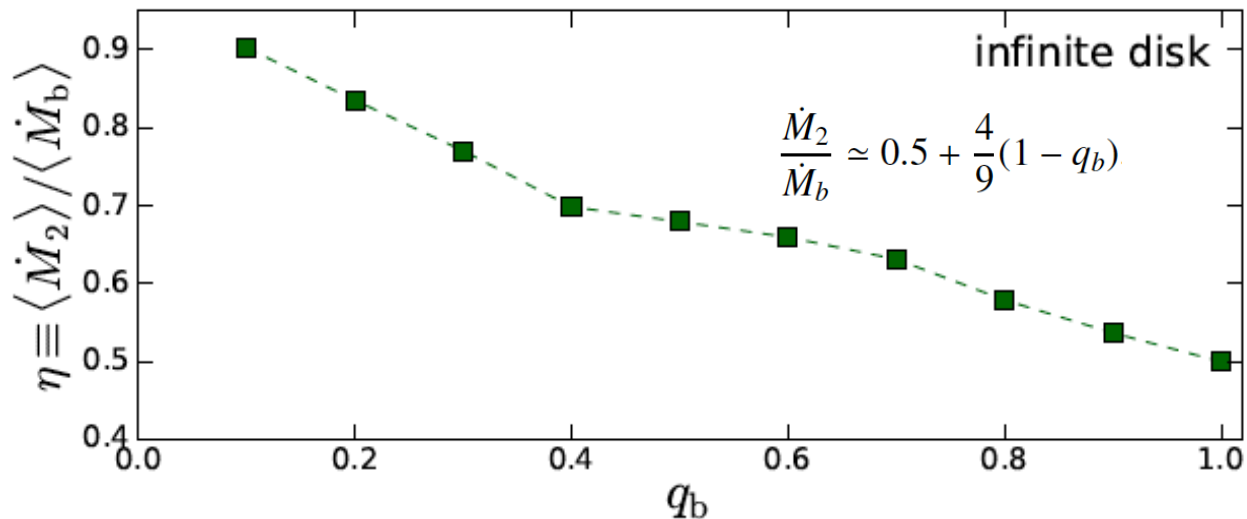
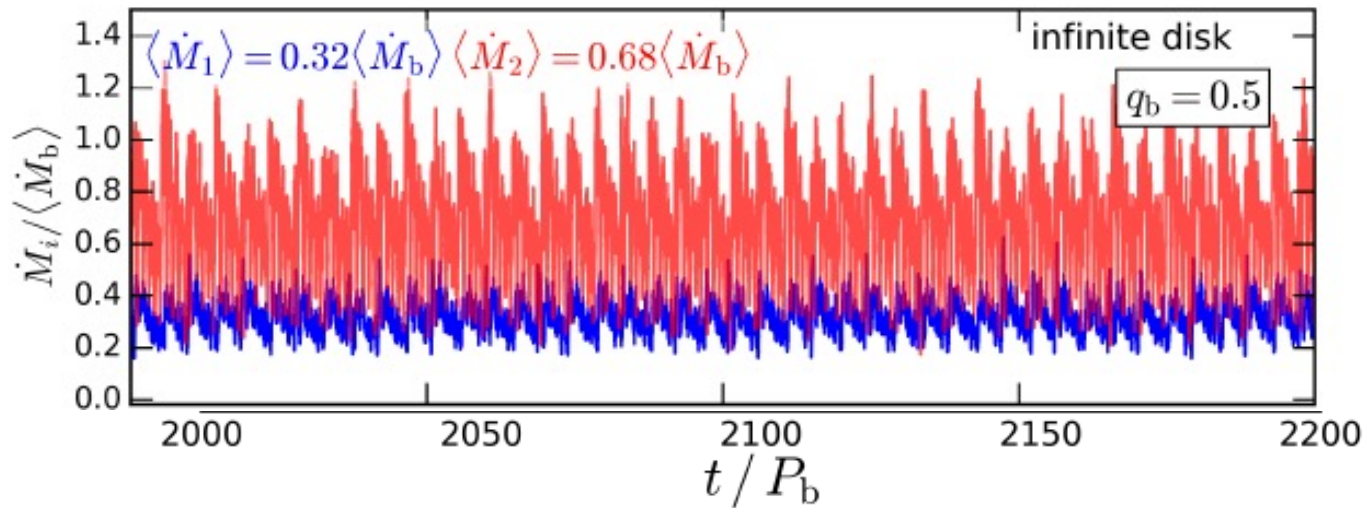
Unequal-mass binaries

$$q = M_2/M_1 < 1$$

$$e_b = 0 \quad \text{Munoz, Lai et al 2020}$$

-- Low-mass component accretes more

See also Bate+2000; Farris+2014



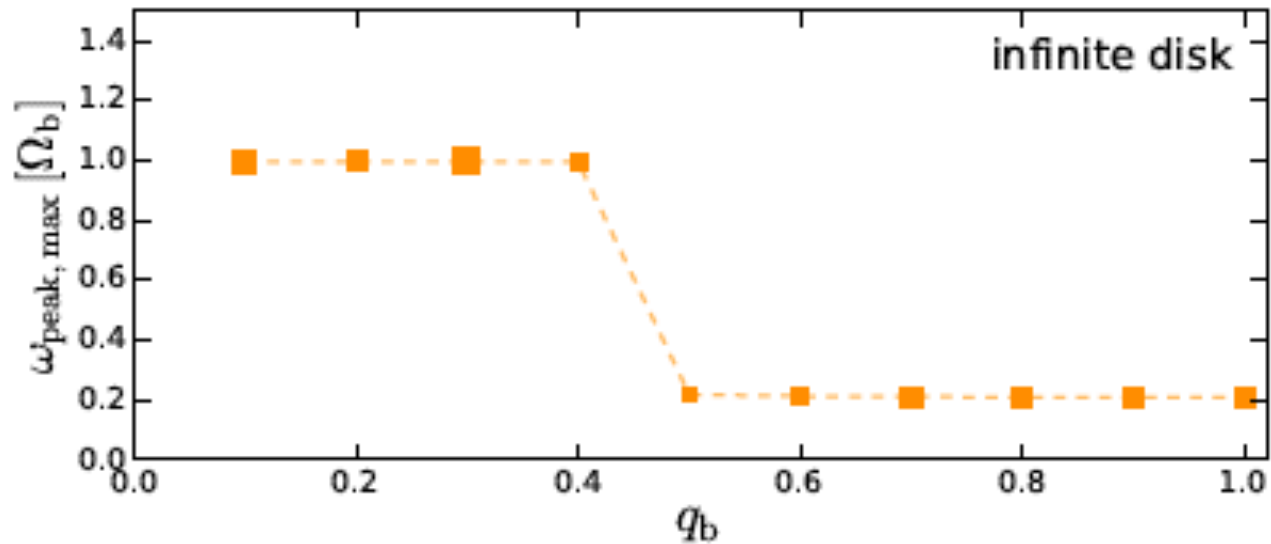
Unequal-mass binaries

$$q = M_2/M_1 < 1$$

$$e_b = 0$$

Munoz, DL +2020

-- Dominant variability frequency



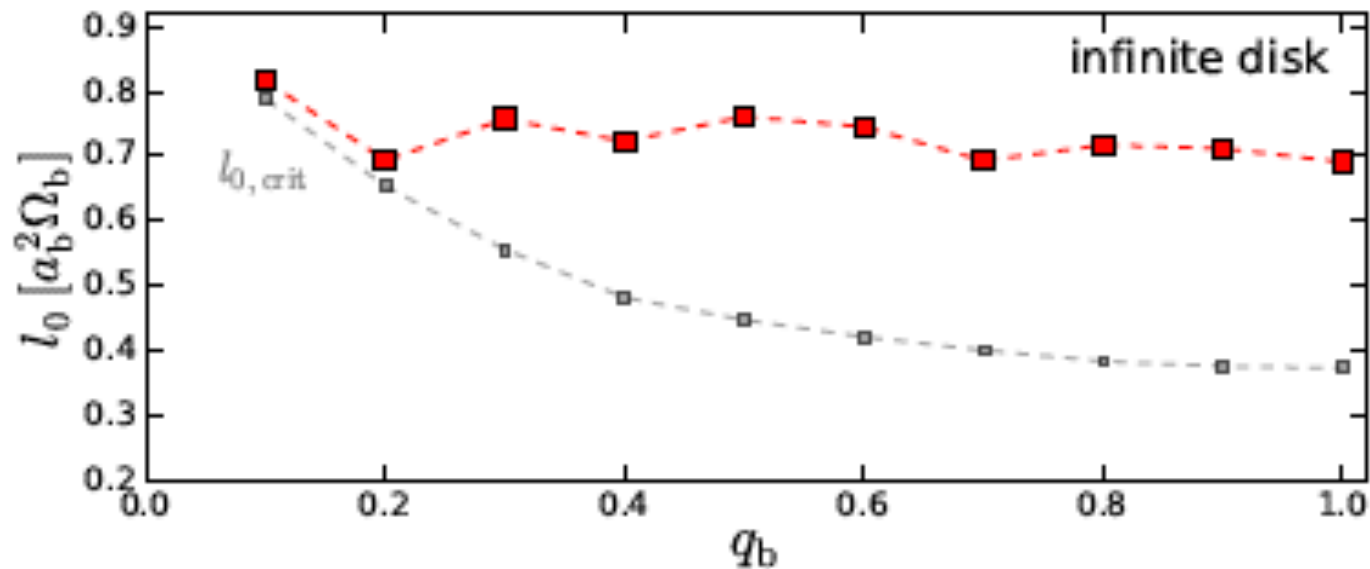
Unequal-mass binaries

$$q = M_2/M_1 < 1$$

$$e_b = 0$$

Munoz, DL +2020

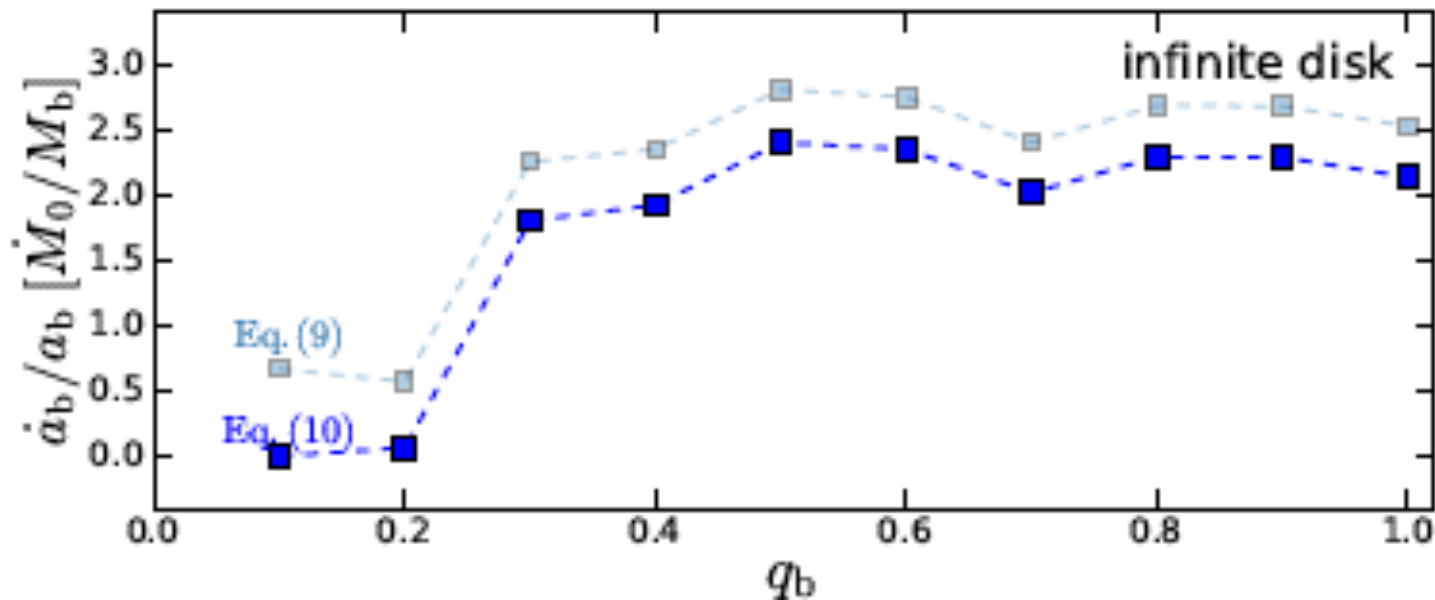
-- Angular momentum transfer



Unequal-mass binaries

$$q = M_2/M_1 < 1$$
$$e_b = 0$$

-- Orbit evolution



Munoz, DL +2020

See also Duffell et al. 2020: $\dot{a}_b < 0$ for $q_b \lesssim 0.05$

Unequal-mass, eccentric binaries:

Siwek et al. 2023

Recap:

In quasi-steady state, comparable-mass binary **can** expand while accreting from CBD

Eccentricity attractor: ~ 0.4

Is binary decay possible ?

e.g. Supermassive BH Binaries, final pc problem

Is binary decay possible ?

e.g. Supermassive BH Binaries, final pc problem

Yes/maybe...

e.g. Thin (low-viscosity) disks

“steady-state”? finite torus = mass-fed disk? Pressure?

Tiede et al. 2022,2024

e.g. Large (locally) massive disk:

$$\Sigma \pi a_b^2 \gtrsim M_2$$

e.g. Gas could get ejected in outflow (?)...

Caveats of 2D viscous hydro simulations:

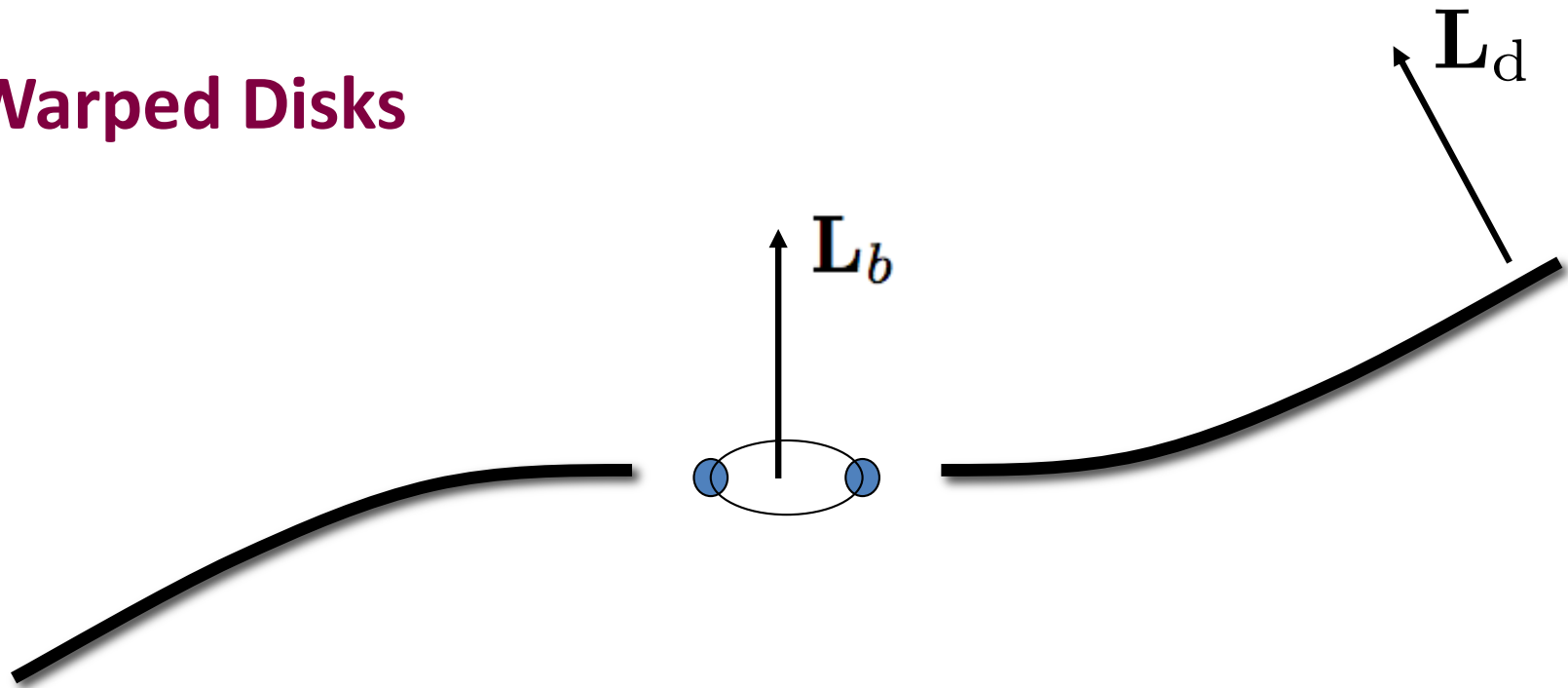
Equation of state/cooling (Haiyang Wang et al 2023)

B fields, turbulence.....

So far: Co-planar disks

What about misaligned disks ?

Warped Disks

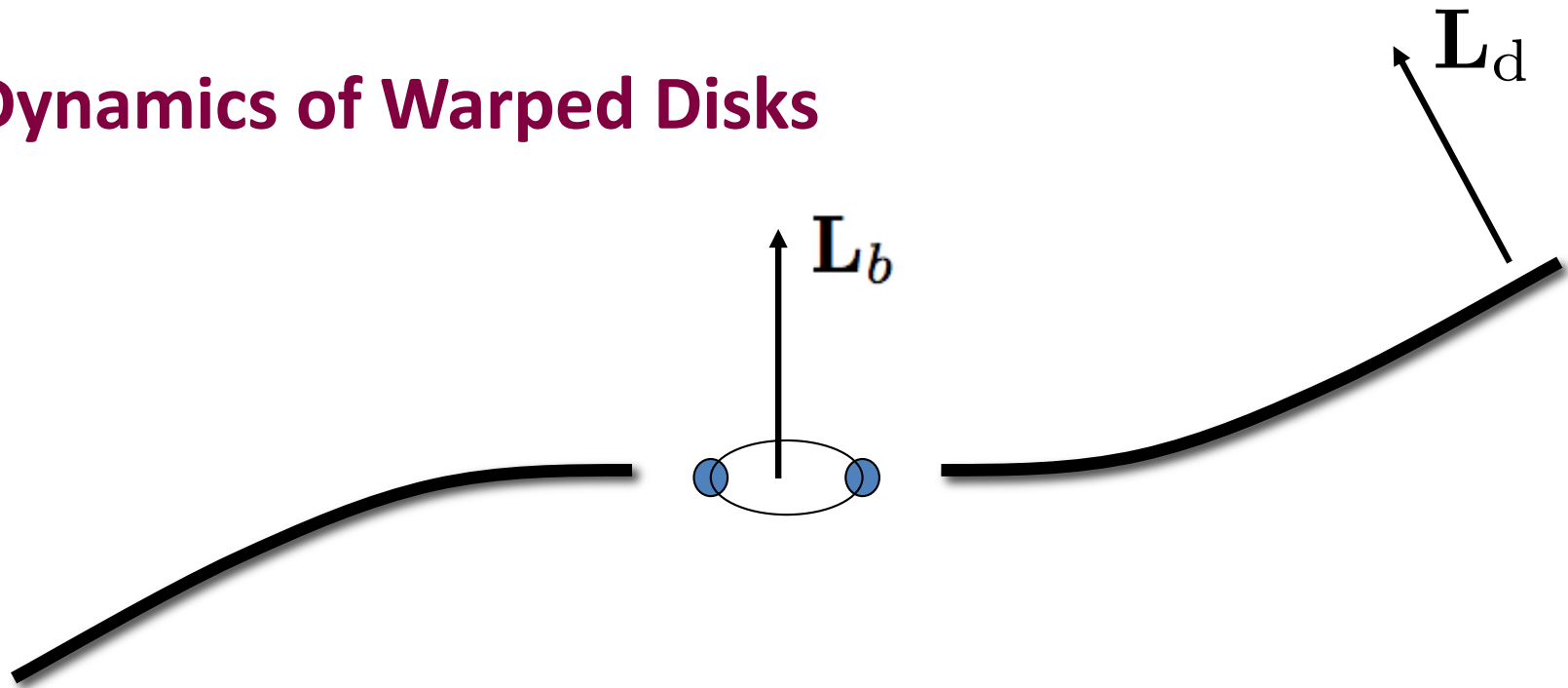


Torque from binary on disk $\Rightarrow$ disk (ring) nodal precession

$$\Omega_p(r) \simeq \frac{3\mu}{4M_t} \left(\frac{a}{r}\right)^2 \Omega(r)$$

Differential precession + internal fluid stress $\Rightarrow$ warped/twisted disk

Dynamics of Warped Disks



Warp + Viscosity $\Rightarrow$ Dissipation $\Rightarrow$ Align $\mathbf{L}_b$ and $\mathbf{L}_d$

$$\frac{\partial \hat{\mathbf{l}}}{\partial \ln r} \sim \frac{\alpha}{c_s^2} \mathbf{T}_{\text{ext}} \quad |\mathbf{T}_{\text{ext}}| \sim r^2 \Omega \omega_{\text{ext}}, \quad \omega_{\text{ext}} = \Omega_{\text{prec}}$$
$$\left| \frac{d\hat{\mathbf{l}}}{dt} \right|_{\text{visc}} \sim \left\langle \left(\frac{\alpha}{c_s^2} \right) \frac{\mathbf{T}_{\text{ext}}^2}{r^2 \Omega} \right\rangle \sim \left\langle \frac{\alpha}{c_s^2} (r^2 \Omega) \omega_{\text{ext}}^2 \right\rangle$$

**Typical alignment time can be short
($\sim$ precession period)**

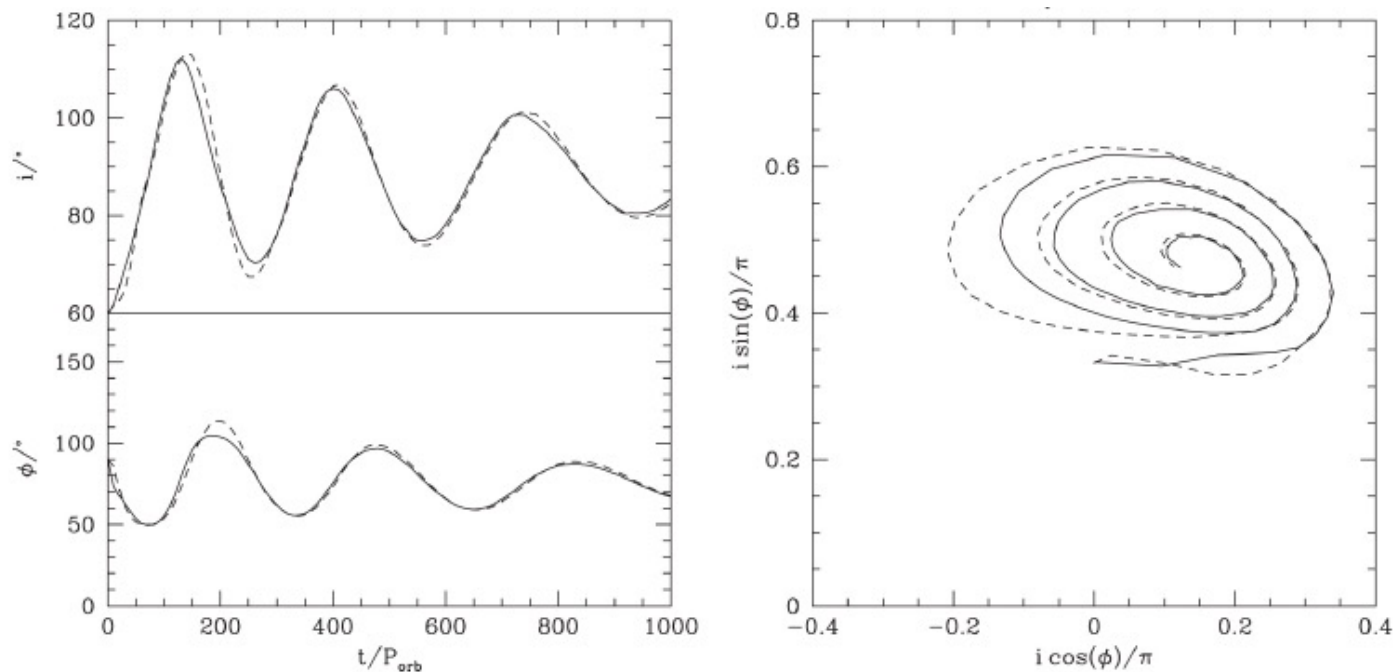
Foucart & DL 2014
Zanazzi & DL 2018

Surprise: Disk around eccentric binary may evolve toward polar alignment

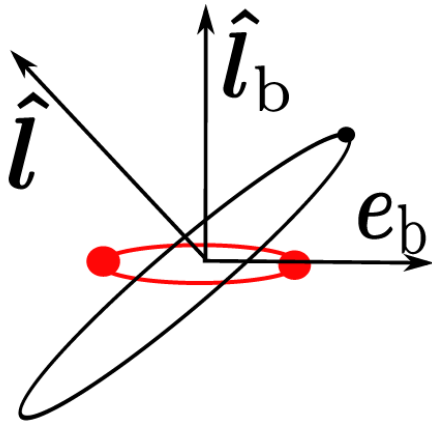
Martin & Lubow (2017): viscous hydro simulation using SPH

Initial disk-binary inclination $i(0) = 60^\circ$

Binary eccentricity $e_b = 0.5$.

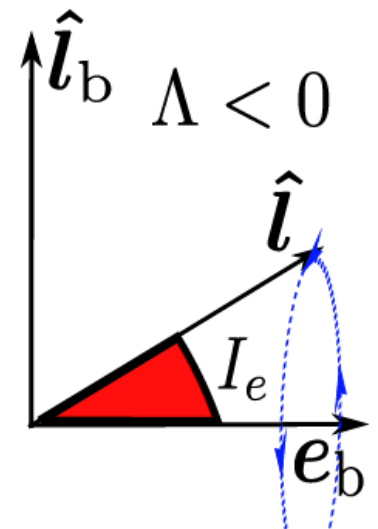
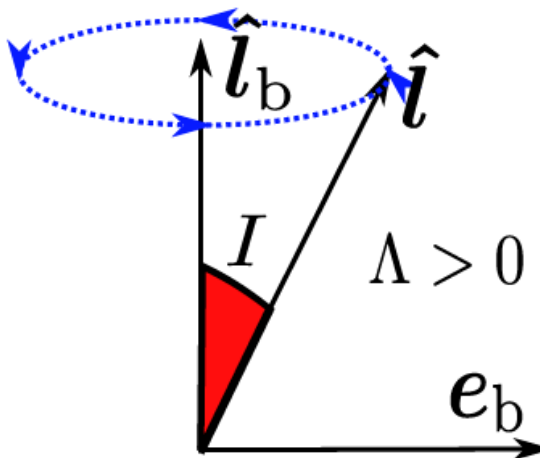


Theoretical Understanding of Polar Alignment of Disks Around Eccentric Binaries

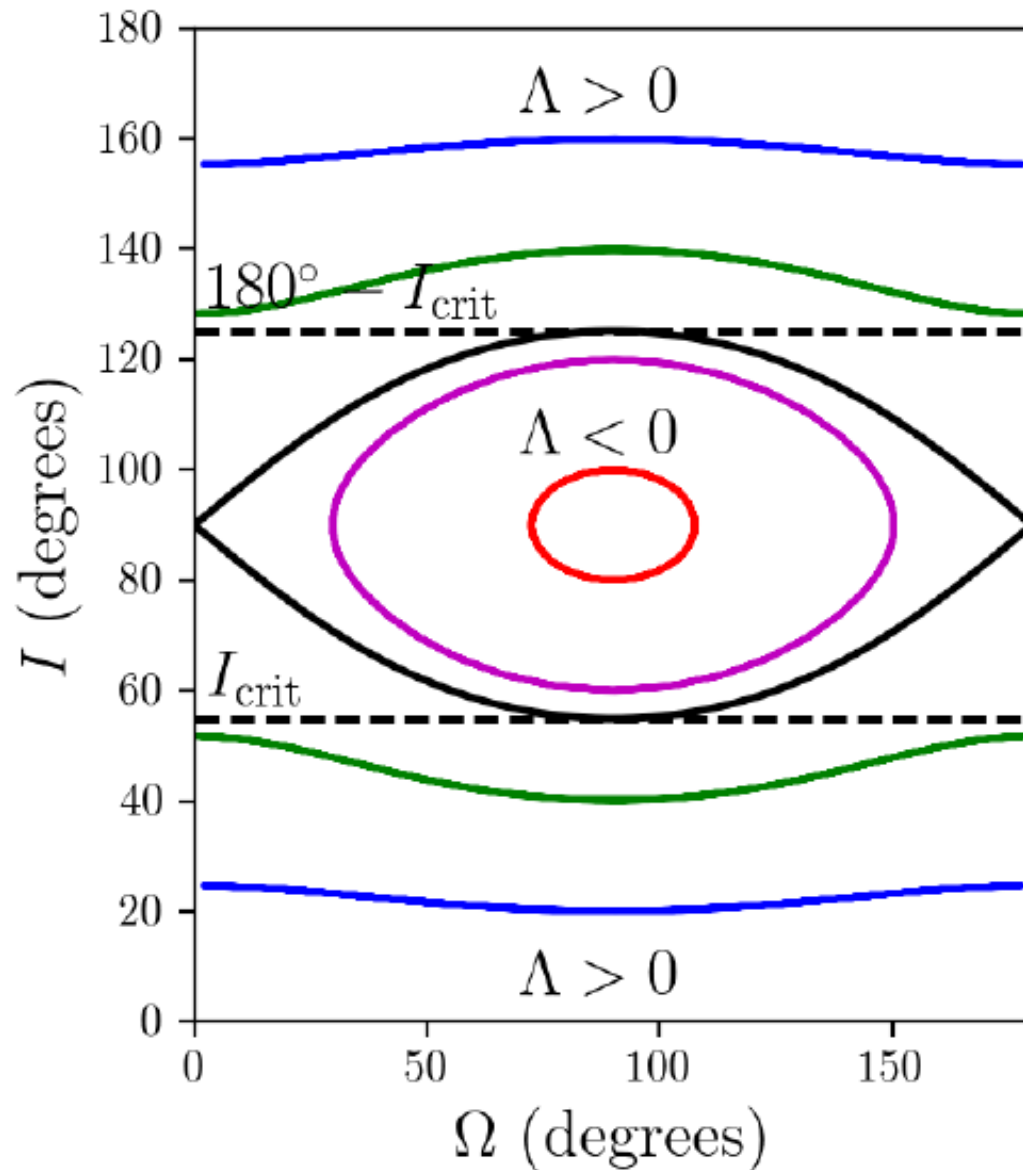


Test particle around eccentric binary has two “masters”

$$\Lambda = (1 - e_b^2)(\hat{l} \cdot \hat{l}_b)^2 - 5(\hat{l} \cdot e_b)^2$$



$$\Lambda = (1 - e_b^2)(\hat{l} \cdot \hat{l}_b)^2 - 5(\hat{l} \cdot \mathbf{e}_b)^2$$

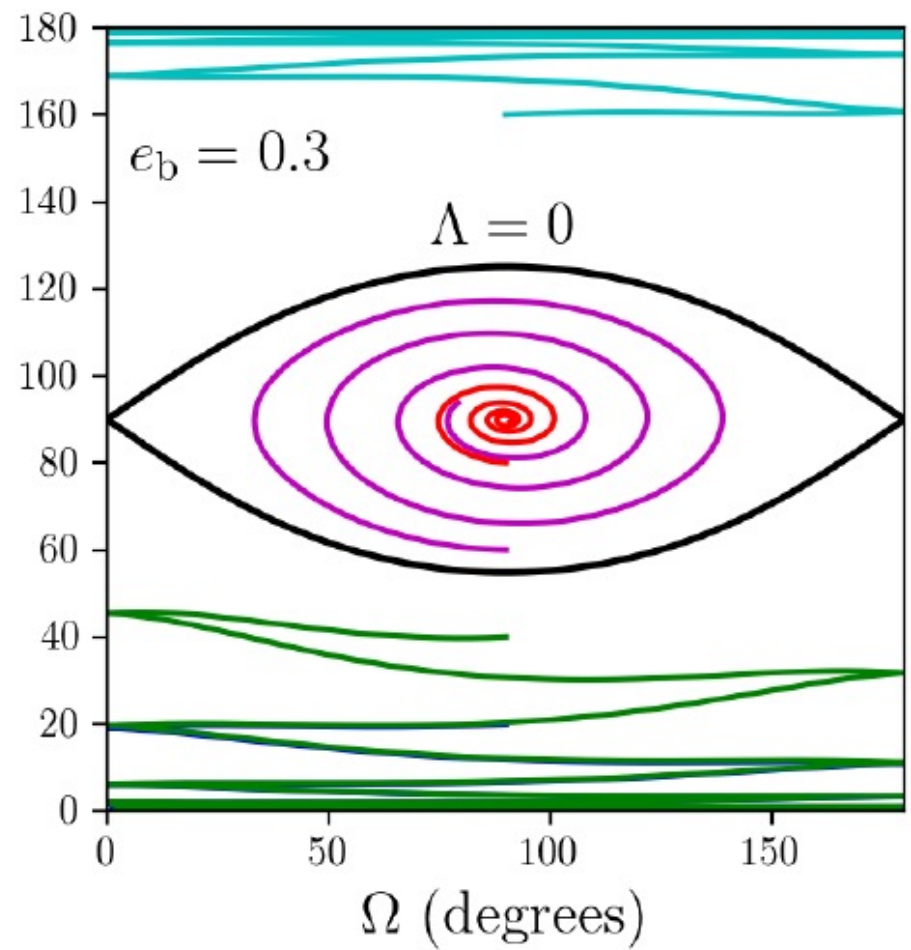
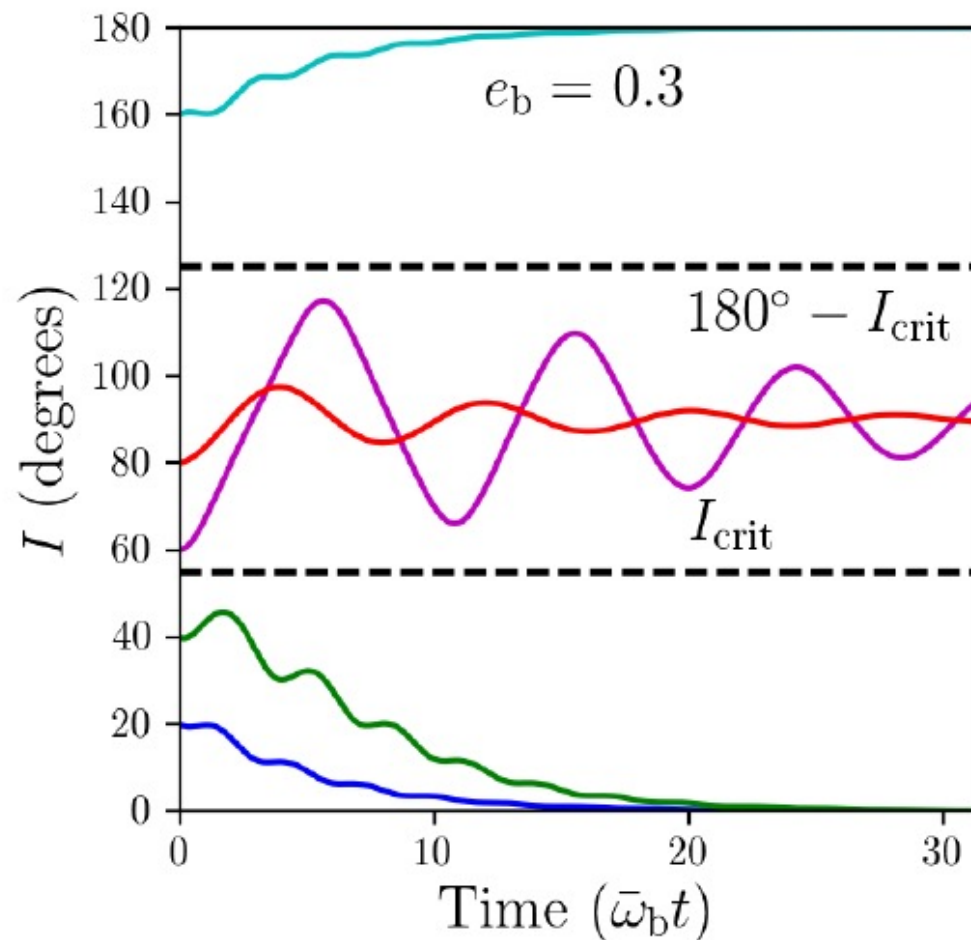


For $\hat{l}$ to precess around $\hat{e}_b$,
require $\sin I > \sin I_{\text{crit}}$

$$I_{\text{crit}} = \cos^{-1} \sqrt{\frac{5e_b^2}{1 + 4e_b^2}}$$

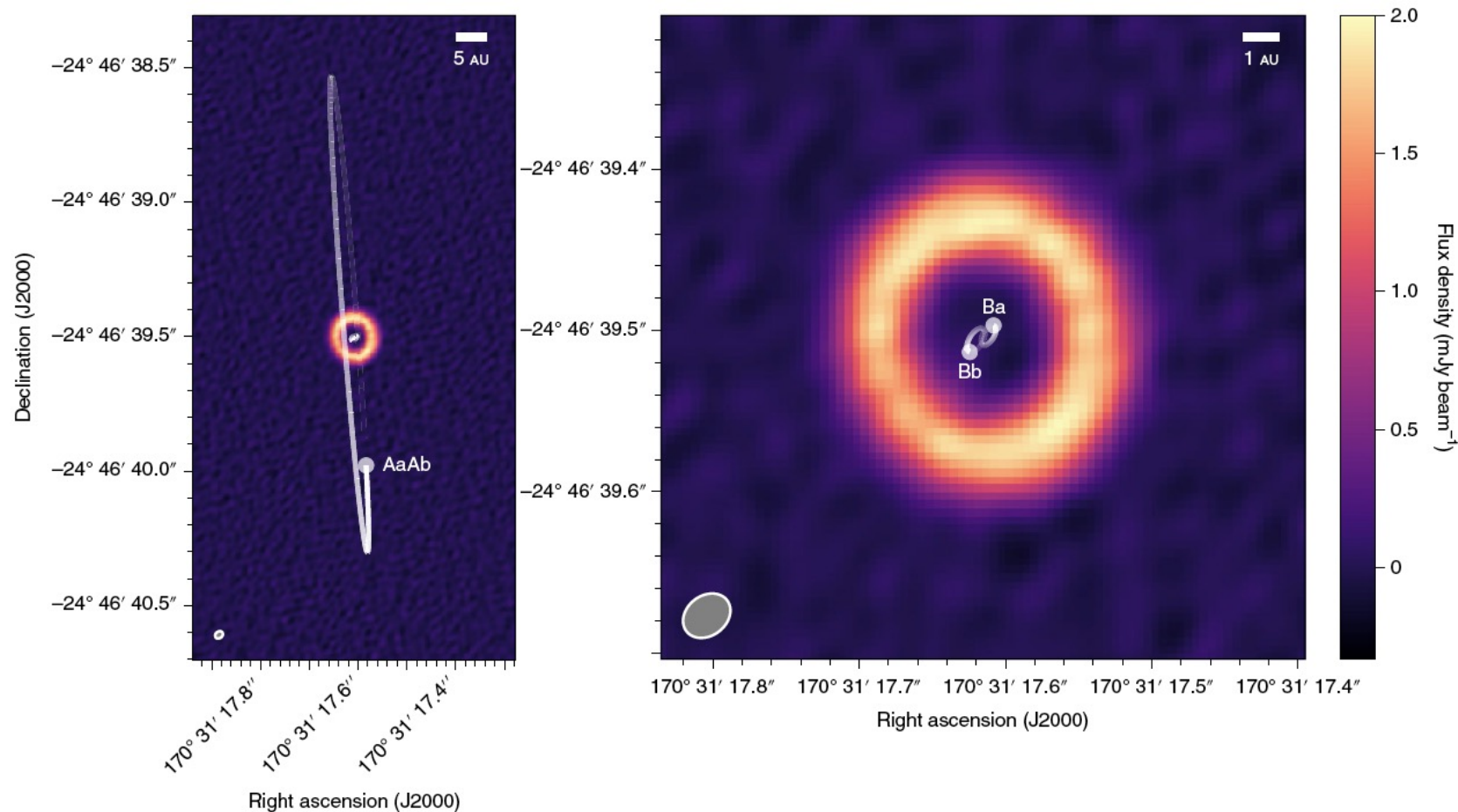
Warped viscous disk around eccentric binary

Evolve towards either align (anti-align) or polar align with the binary

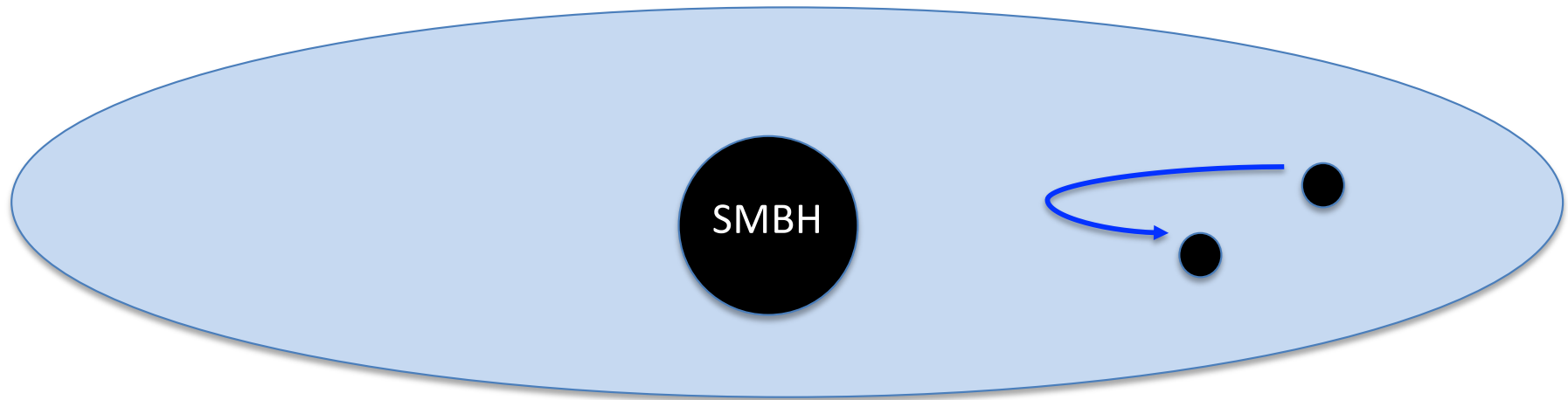


A circumbinary protoplanetary disk in a polar configuration

Grant M. Kennedy^{1,2*}, Luca Matrà³, Stefano Facchini^{4,5}, Julien Milli⁶, Olja Panić⁷, Daniel Price^{8,9}, David J. Wilner³, Mark C. Wyatt¹⁰ and Ben M. Yelverton¹⁰



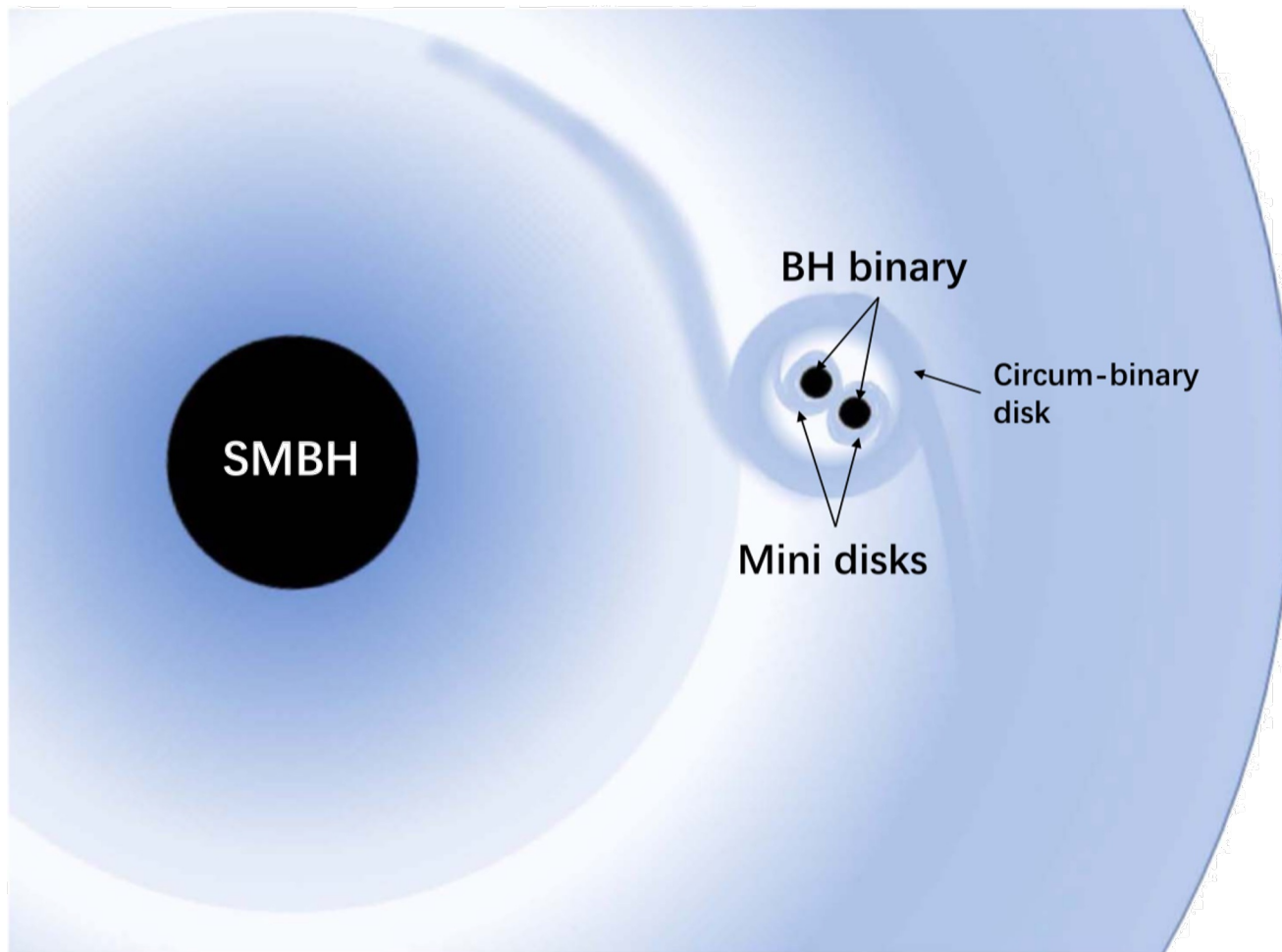
Small Binary embedded in ANG Disk



Binary BH mergers in AGN disks ?

McKernan+12, Bellovary+16, Bartos+17, Stone+17, McKernan+18, Secunda+18, Yang+19, Tagawa+20, etc

Is it like “Circumbinary Accretion”?



Picture from Li & Cheng (2019)

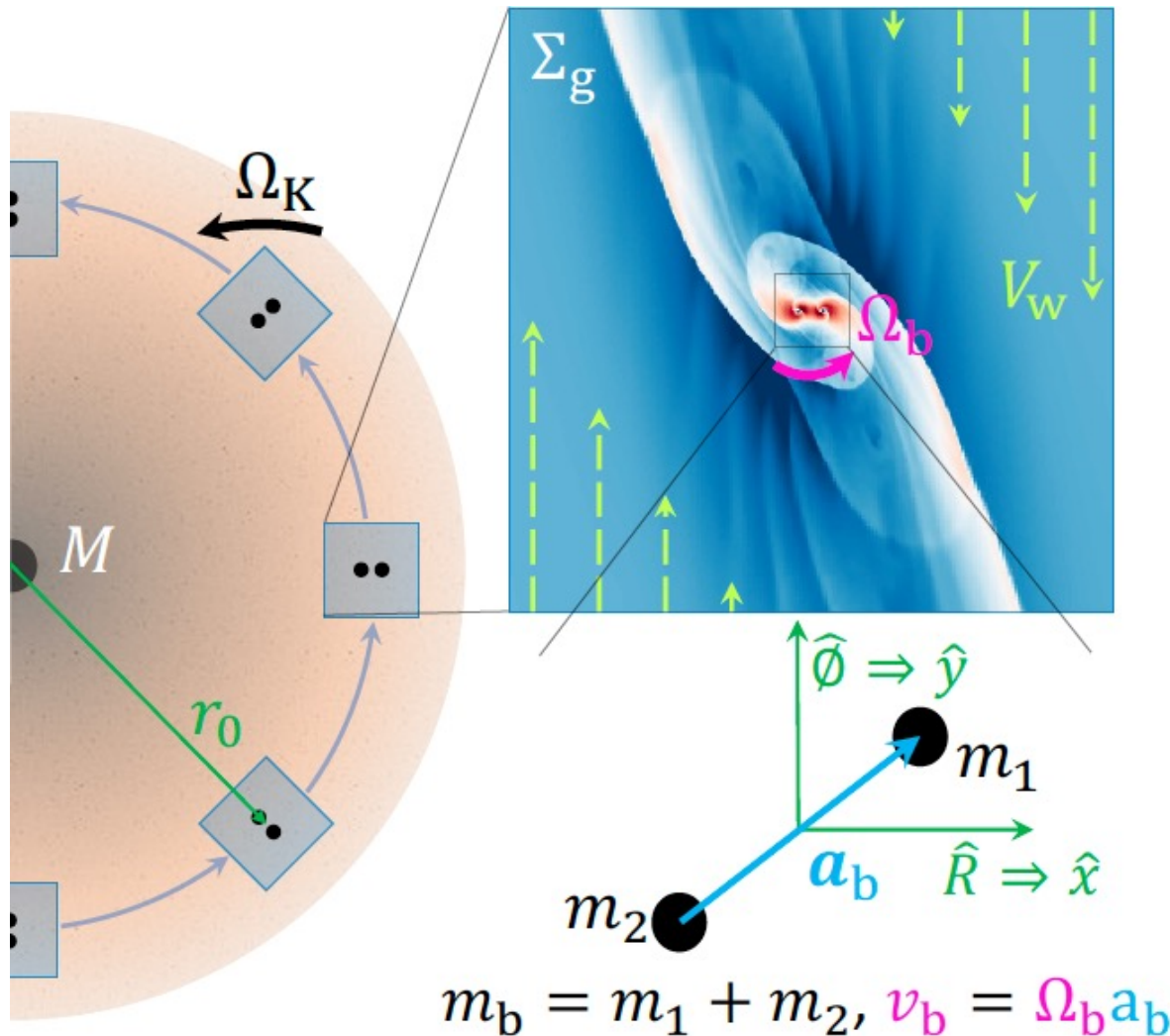
Not clear in general
What is $\dot{M}$?
How does binary evolve?

Numerical studies:

Baruteau et al. 2011
Y.Li,... H.Li... 2021
Dempsey, H.Li... 2023
R.Li & DL 2022,23,24

Local simulations of binary in disk

Rixin Li & Lai 2022,2023,2024

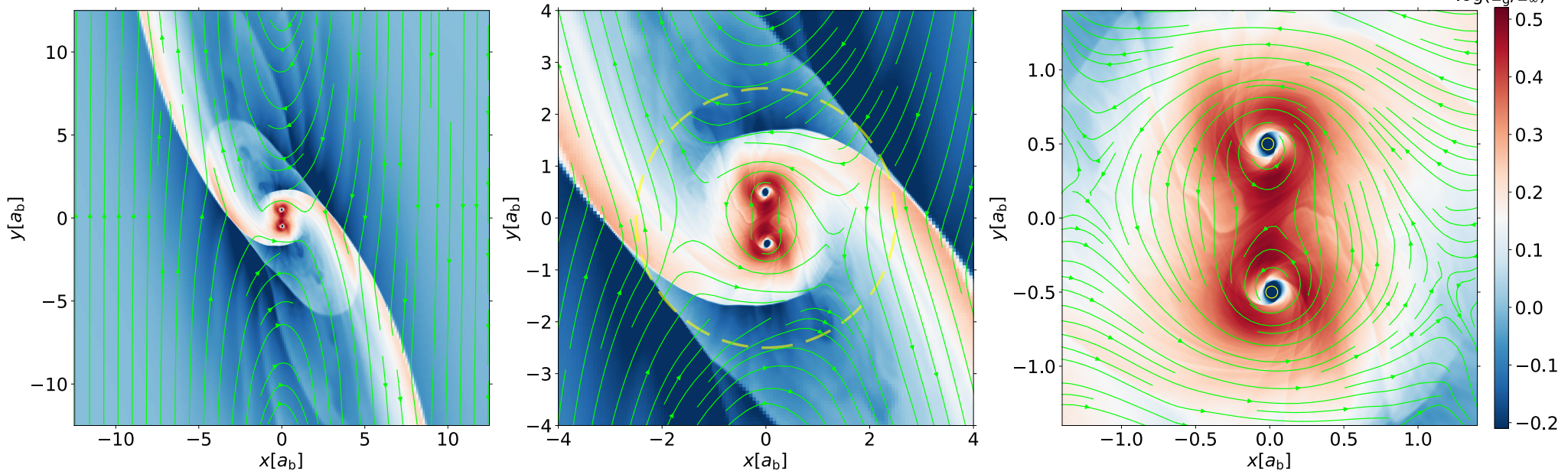


Local shearing box
(not "local wind tunnel box"
used by Kaaz et al. 2021)

ATHENA
Mesh refinement
Resolution: $a_b \sim 250$ cells
zero softening in gravity

Example of flow structure

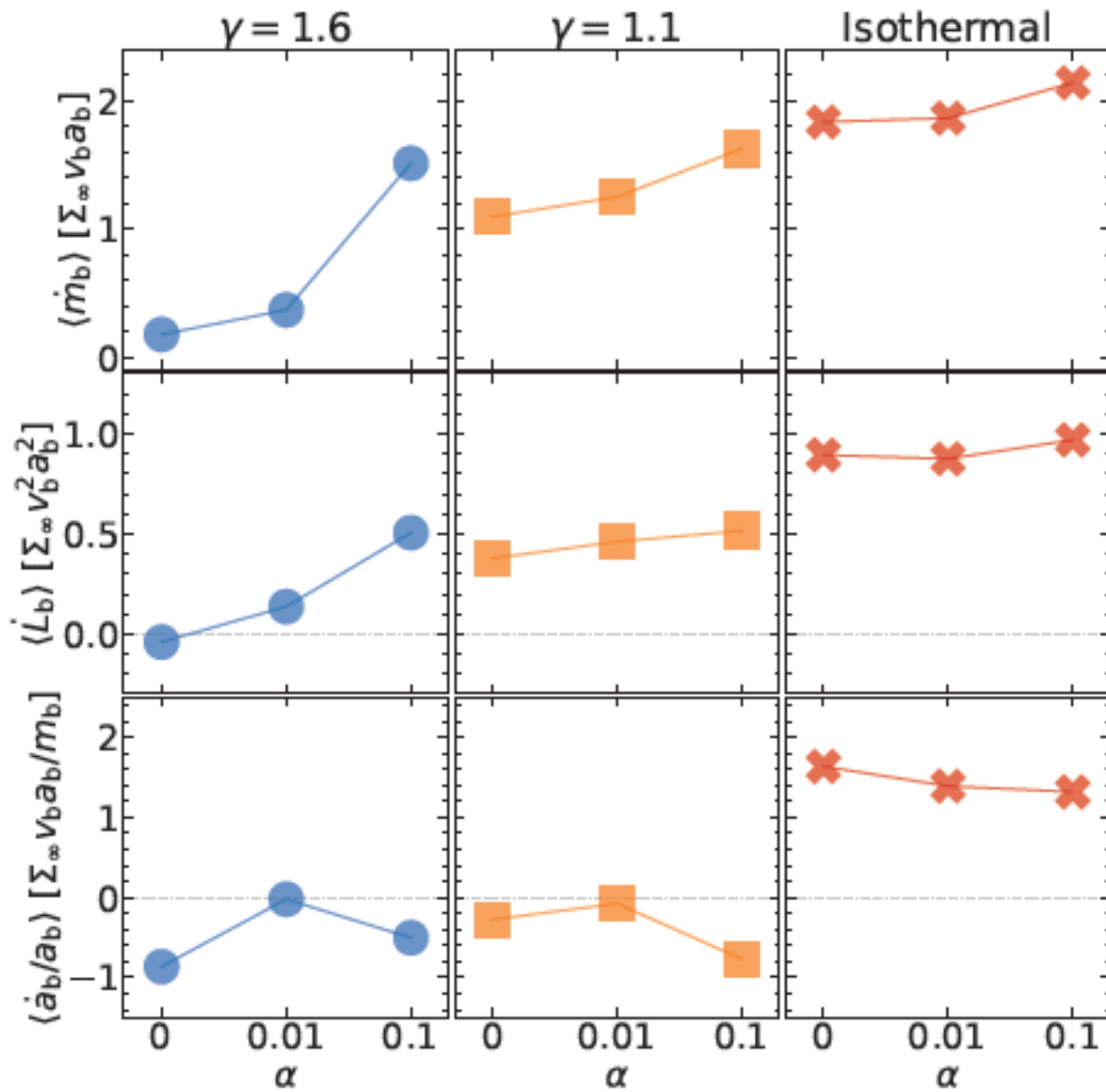
Pairs of bow shocks, spiral shocks



BH = absorbing sphere: sink radius: $r_{\text{sink}} = 0.04 a_b \simeq 10$ cells $\Rightarrow \dot{m}_b$

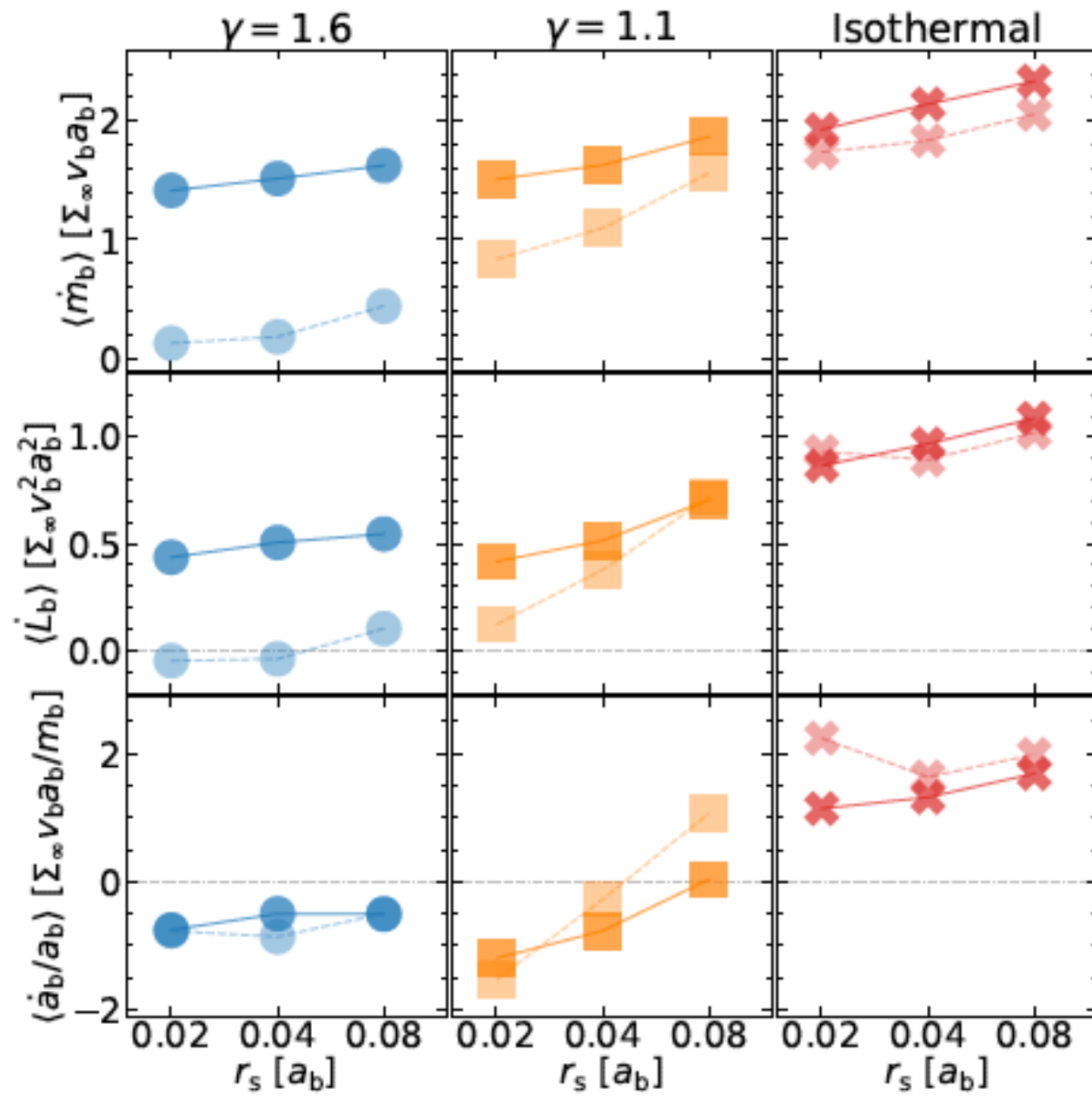
Force on each BH: from gravity + accretion + pressure

$\Rightarrow$ Torque on binary, energy transfer rate $\Rightarrow \dot{a}_b, \dot{e}_b$



Rixin Li & DL 2024

$$\frac{m_1}{m_2} = 1, \quad e_b = 0, \quad \frac{R_H}{H} = \left(\frac{q}{h^3} \right)^{1/3} = 3^{1/3}, \quad \frac{R_H}{a_b} = 5 \quad \text{prograde}$$



Solid line: $\alpha=0.1$
Dashed line: $\alpha=0$

Recap: Embedded binaries in gas disks

Prograde binaries:

Orbital decay or expansion? Depends on gas thermodynamics, viscosity, size of accretor...

Retrograde binaries:

Always orbital decay

In general, orbital evolution of the binary is much faster than migration of the center of mass of binary in AGN disk.

Summary

◆ Understanding circumbinary accretion is

Important: connect to SMBH binaries, PPDs and planets

Challenging: long-term secular effect in the presence of highly dynamical flows

◆ Key Recent Results:

- short-term variabilities: $\sim 5 P_b$ (for $e_b \sim 0$) vs P_b (finite e_b , or $q < 0.4$)
- Small-mass accretes more; symmetry breaking in accretion ($q=1$, finite e_b)
- Binary can gain angular momentum and can expand ($q > 0.1$)
- Eccentricity attractor $e_b \sim 0.4$

◆ Misaligned Disks

- Dissipation leads to either alignment or polar alignment with binary

◆ Binary in a Big Disk

Reference: Lai & Munoz 2023, ARAA +