# Hot Jupiters and Super-Earths:
## Spin-Orbit Dynamics in Exoplanetary Systems

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Michigan State University, March 13, 2025

# Outline: Two topics on spin-orbit misalignments

1. Stellar spin – Planet Orbit (= stellar obliquities)
   in Hot Jupiter systems

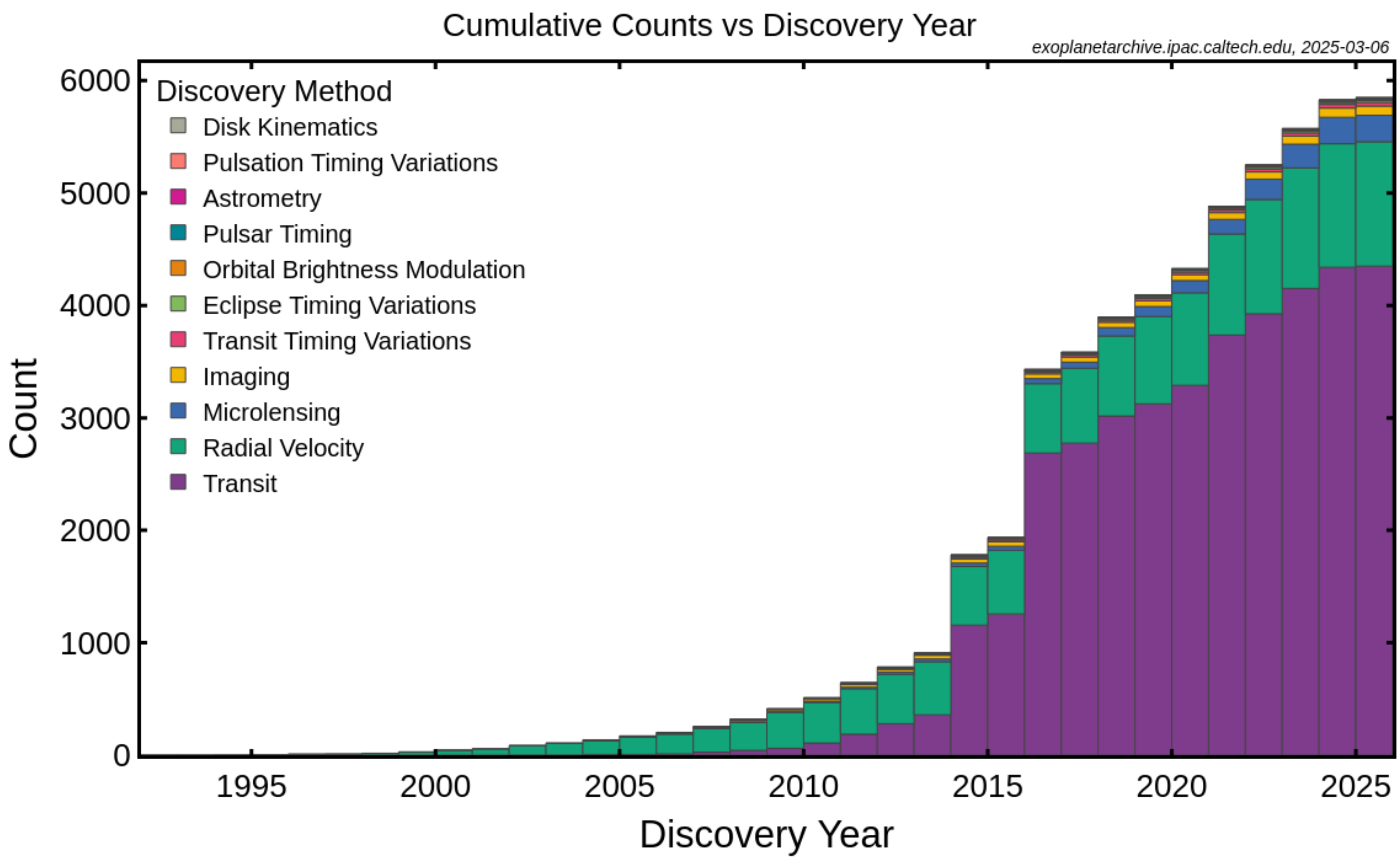
2. Planet spin – Planet Orbit (= planetary obliquities)
   in super-Earth systems

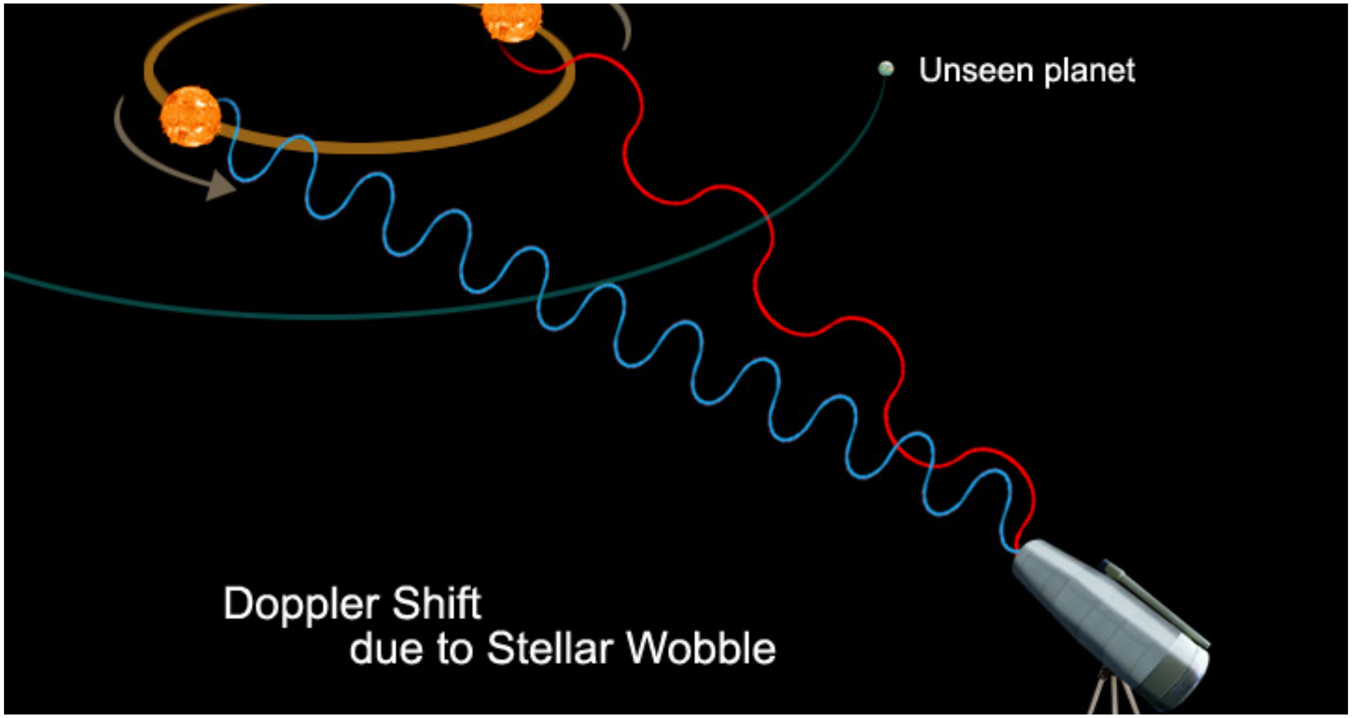
## My own works with

Natalia Storch (Ph.D.16)
JJ Zanazzi (Ph.D.18→CITA→Berkeley)
Kassandra Anderson (Ph.D.19)
Michelle Vick (Ph.D.20→Northwestern→Brown)
Yubo Su (Ph.D.22→Princeton)
Jiaru Li (Ph.D.23→Northwestern)

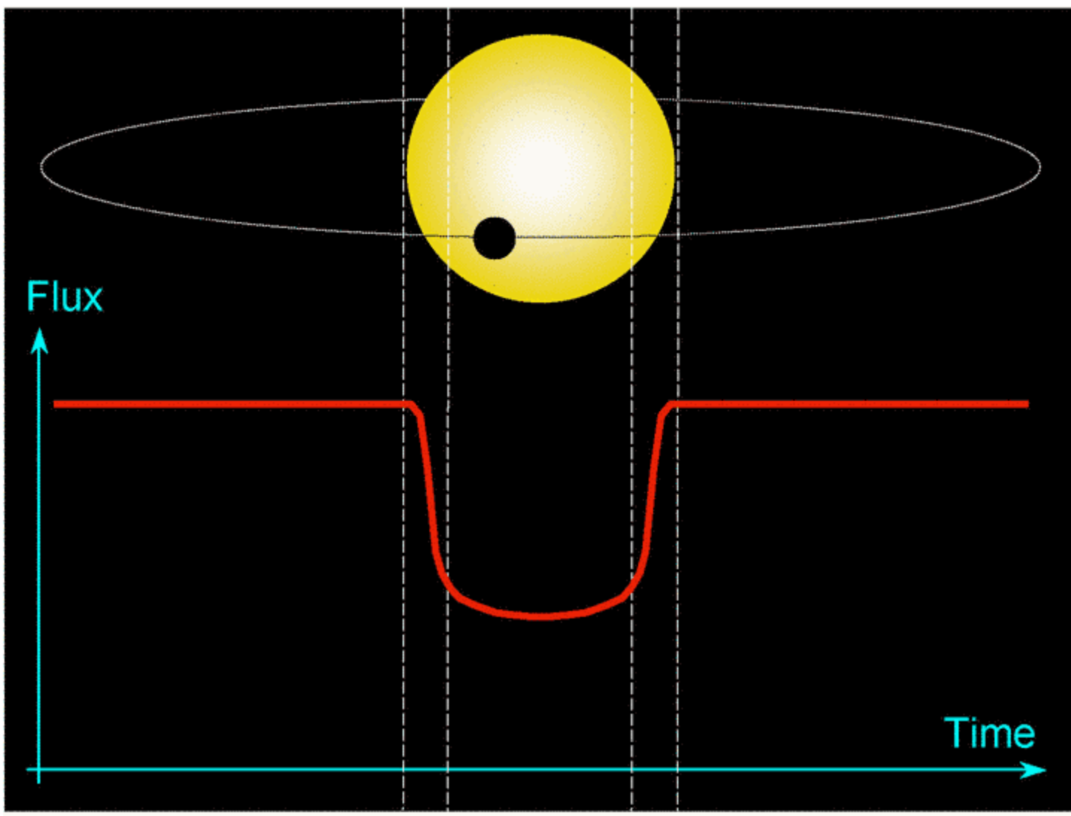
Other related works by: A. Correia, J. Laskar, D. Fabrycky, K. Batygin, S. Millholland ….

# Number of exoplanets discovered: >5000

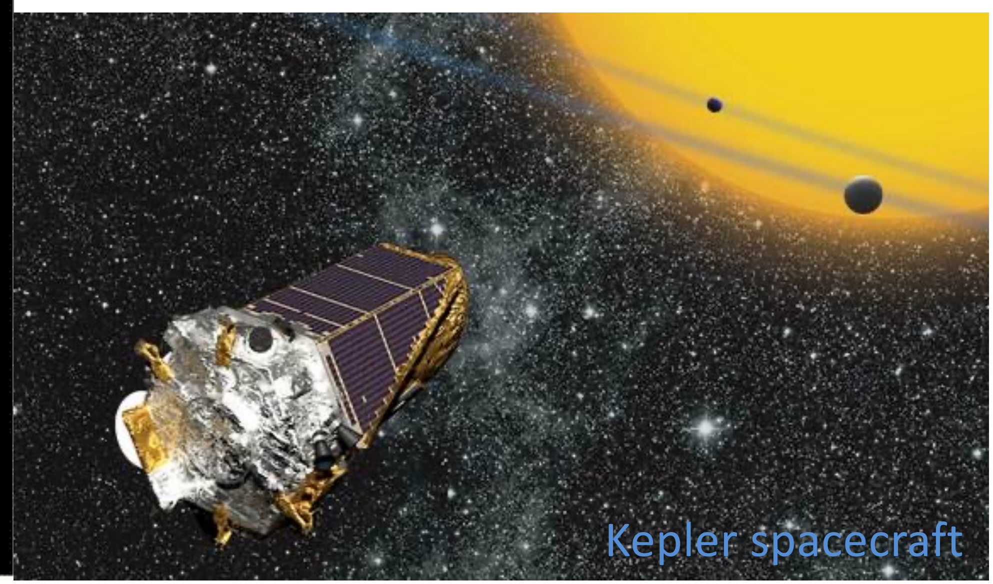




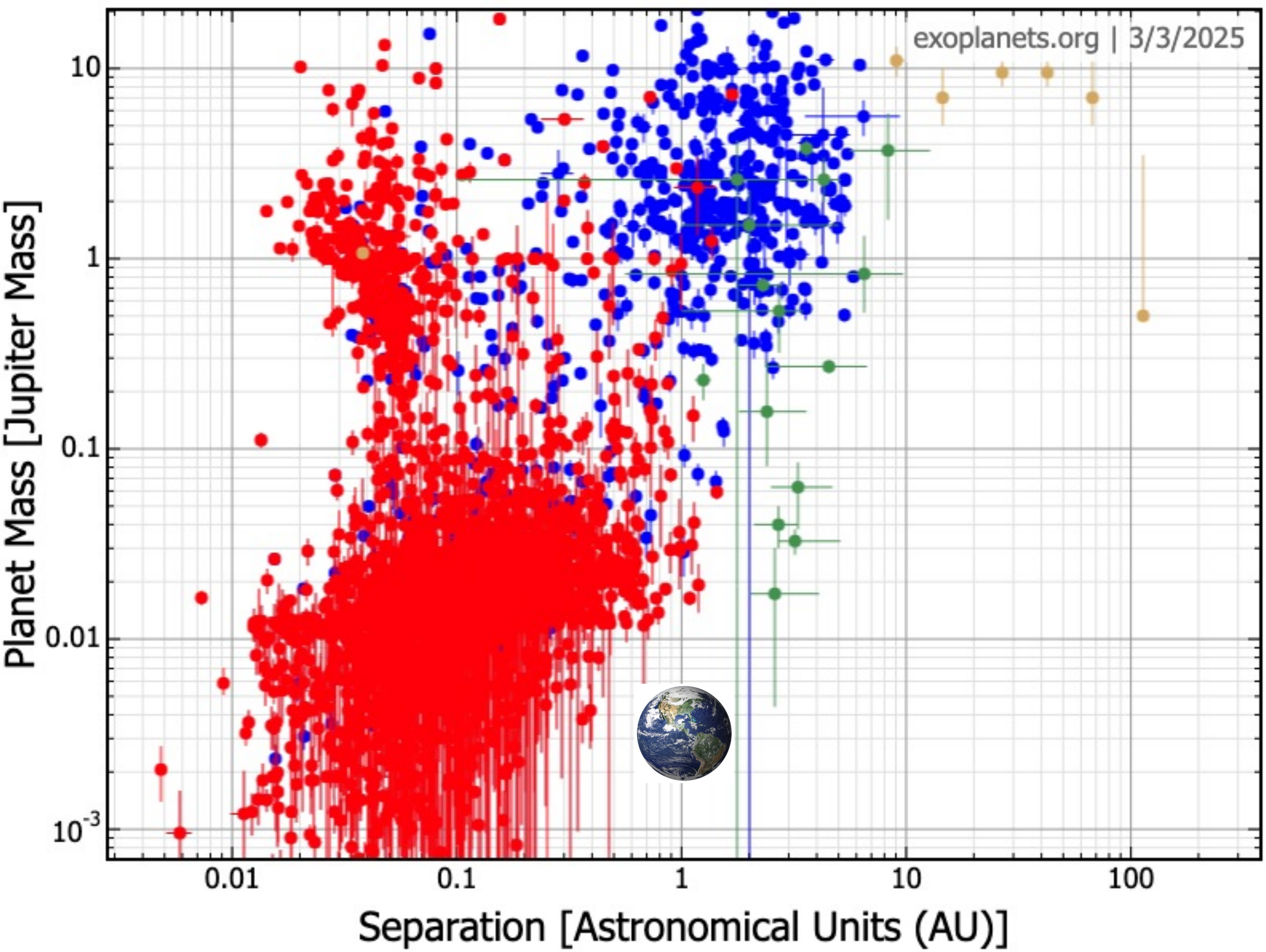

Doppler Method
("Radial Velocity")



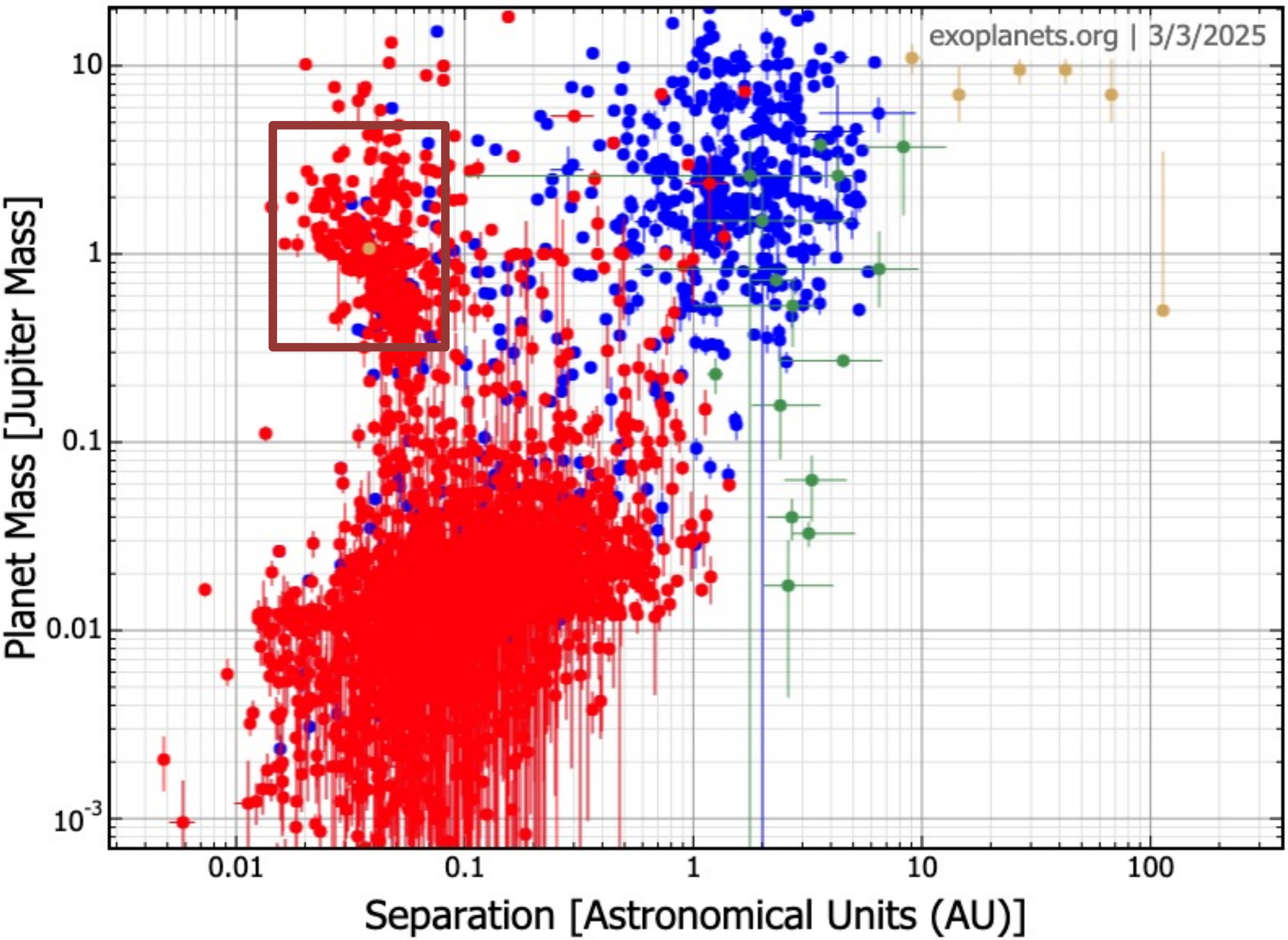

Transit Method



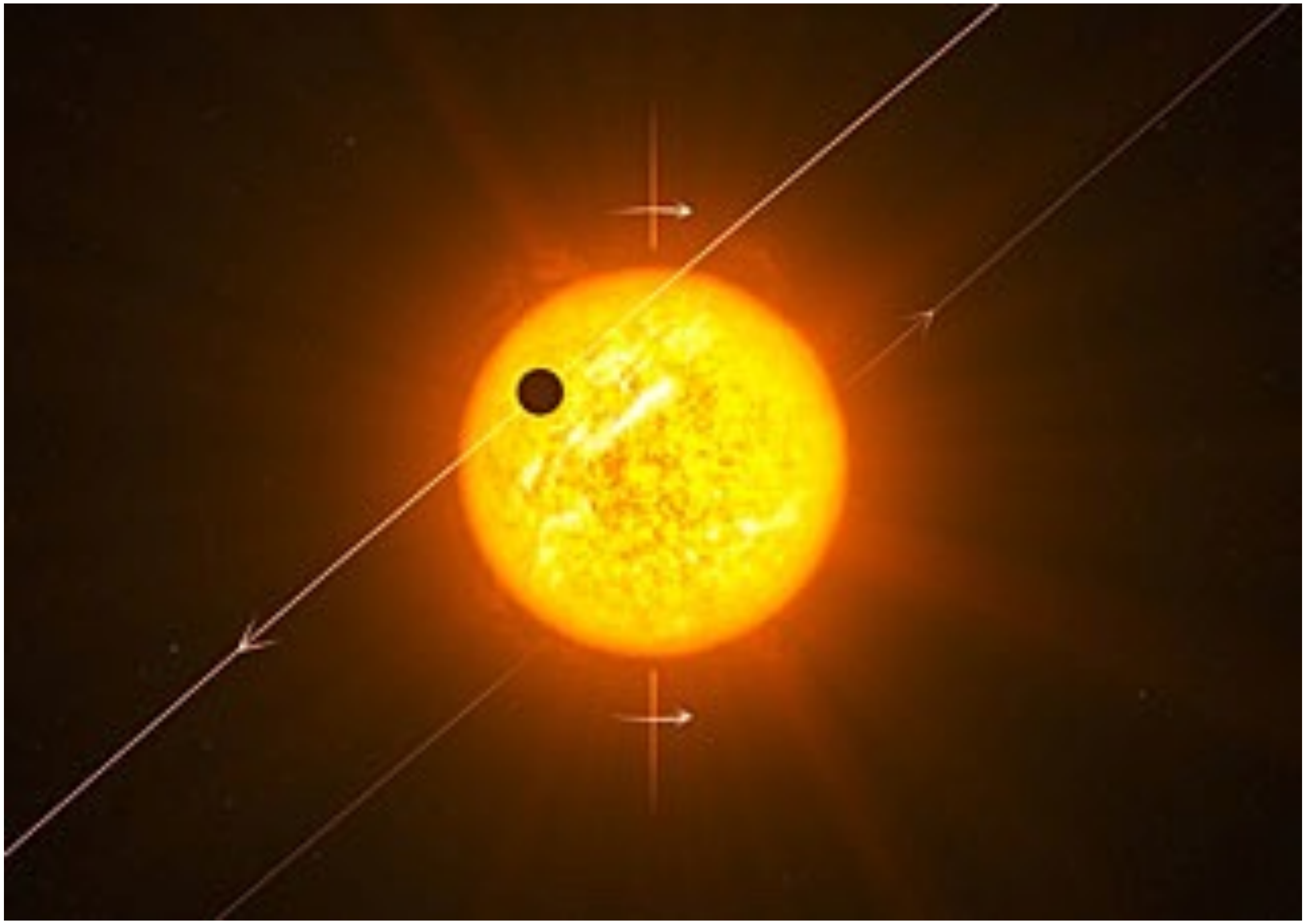
30% of stars in Galaxy have planetary system,
each contains (on average) 3 planets

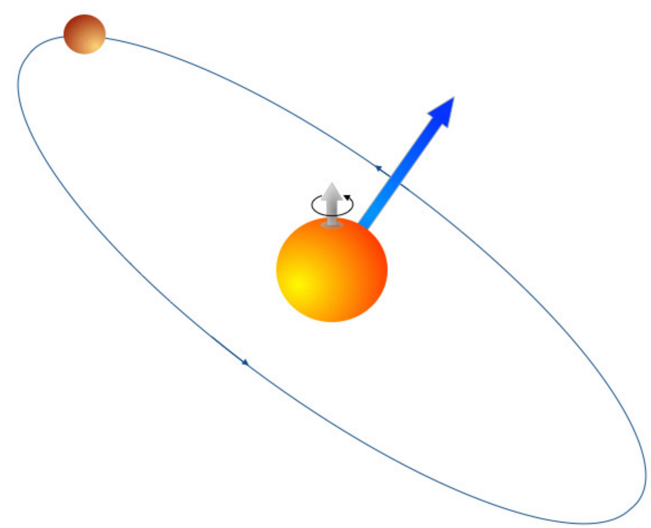

exoplanets.org | 3/3/2025
Planet Mass [Jupiter Mass]
Separation [Astronomical Units (AU)]
10
1
0.1
0.01
10^-3
0.01
0.1
1
10
100

# Hot Jupiters: Giant Planets with P < 10 days



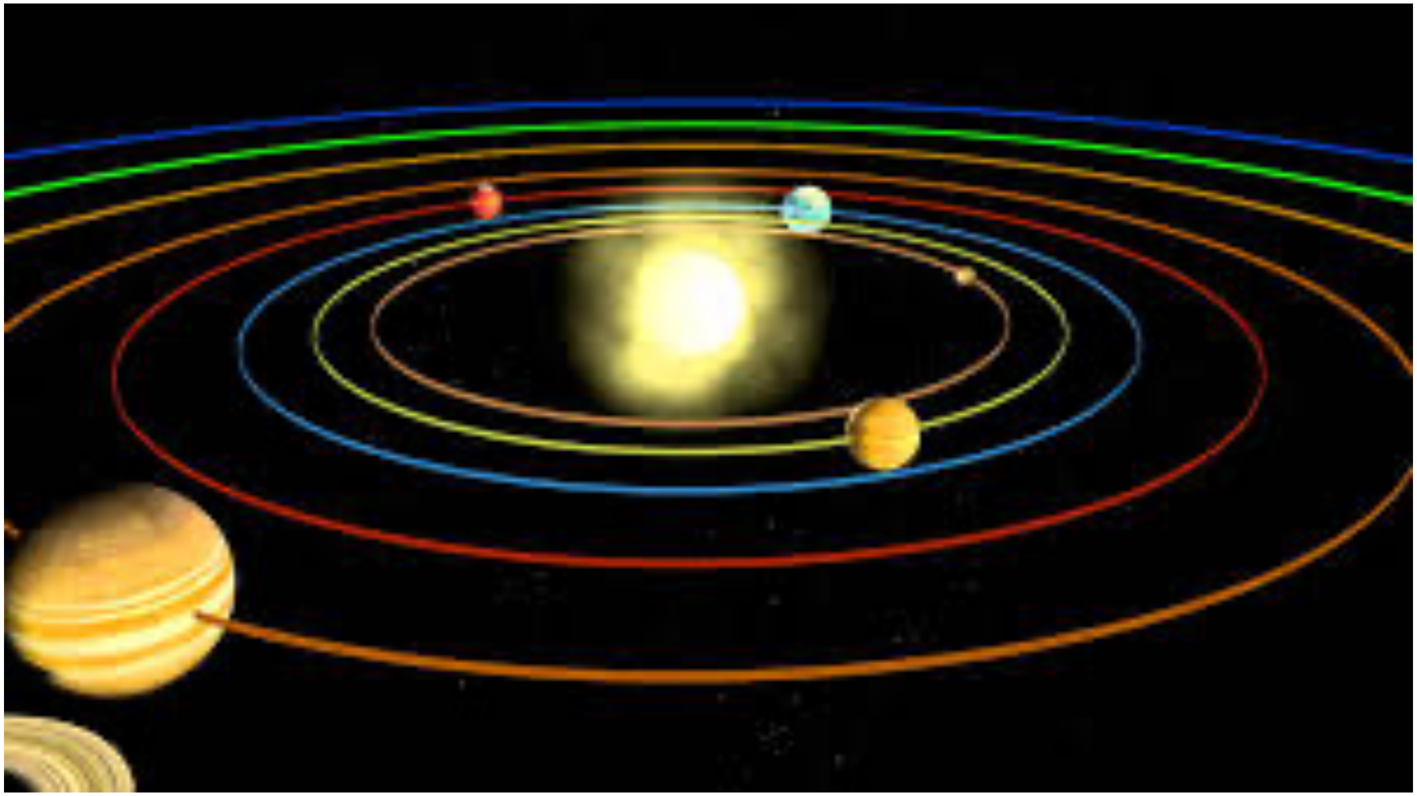
# Spin-Orbit Misalignment Puzzle

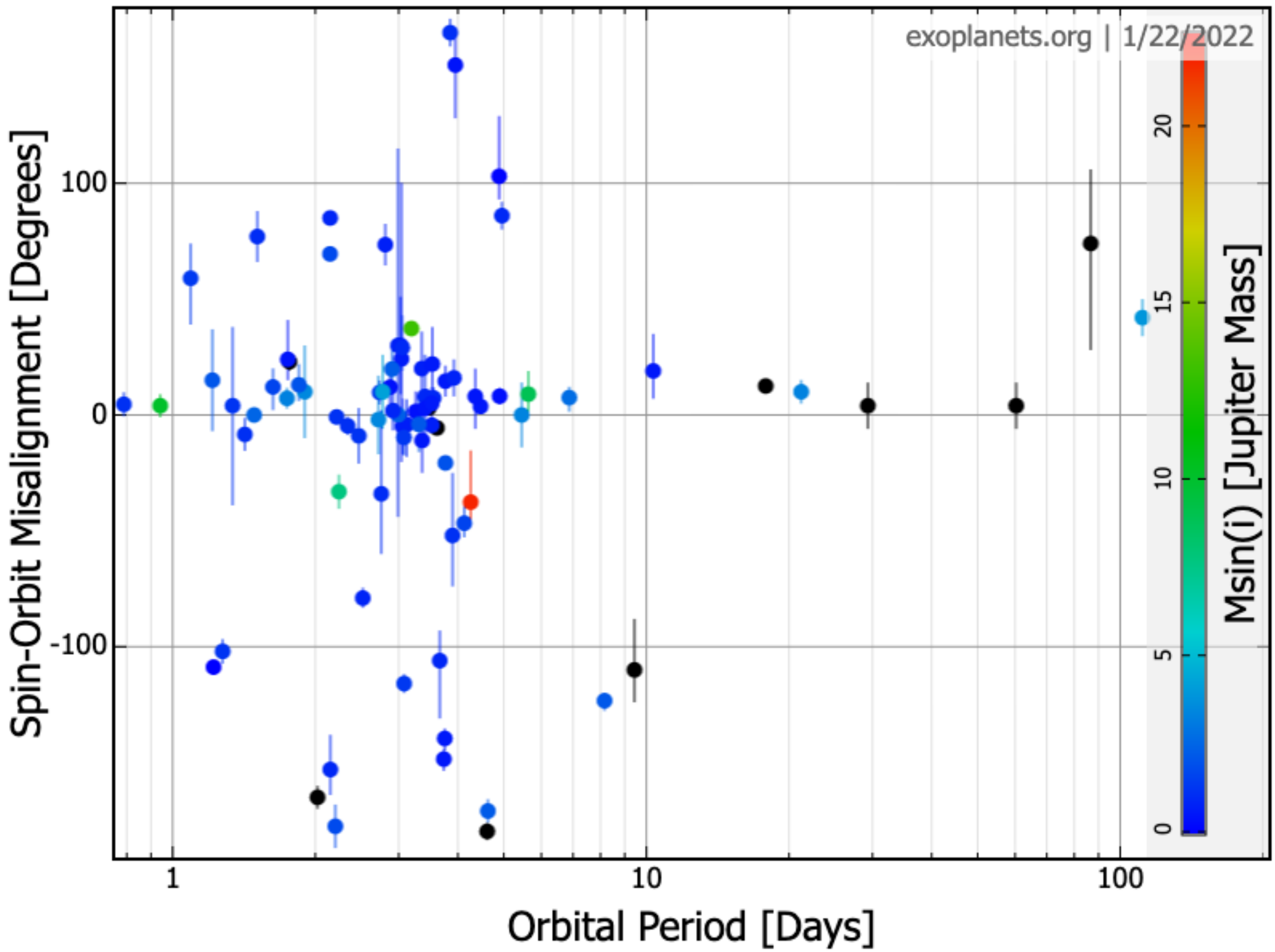


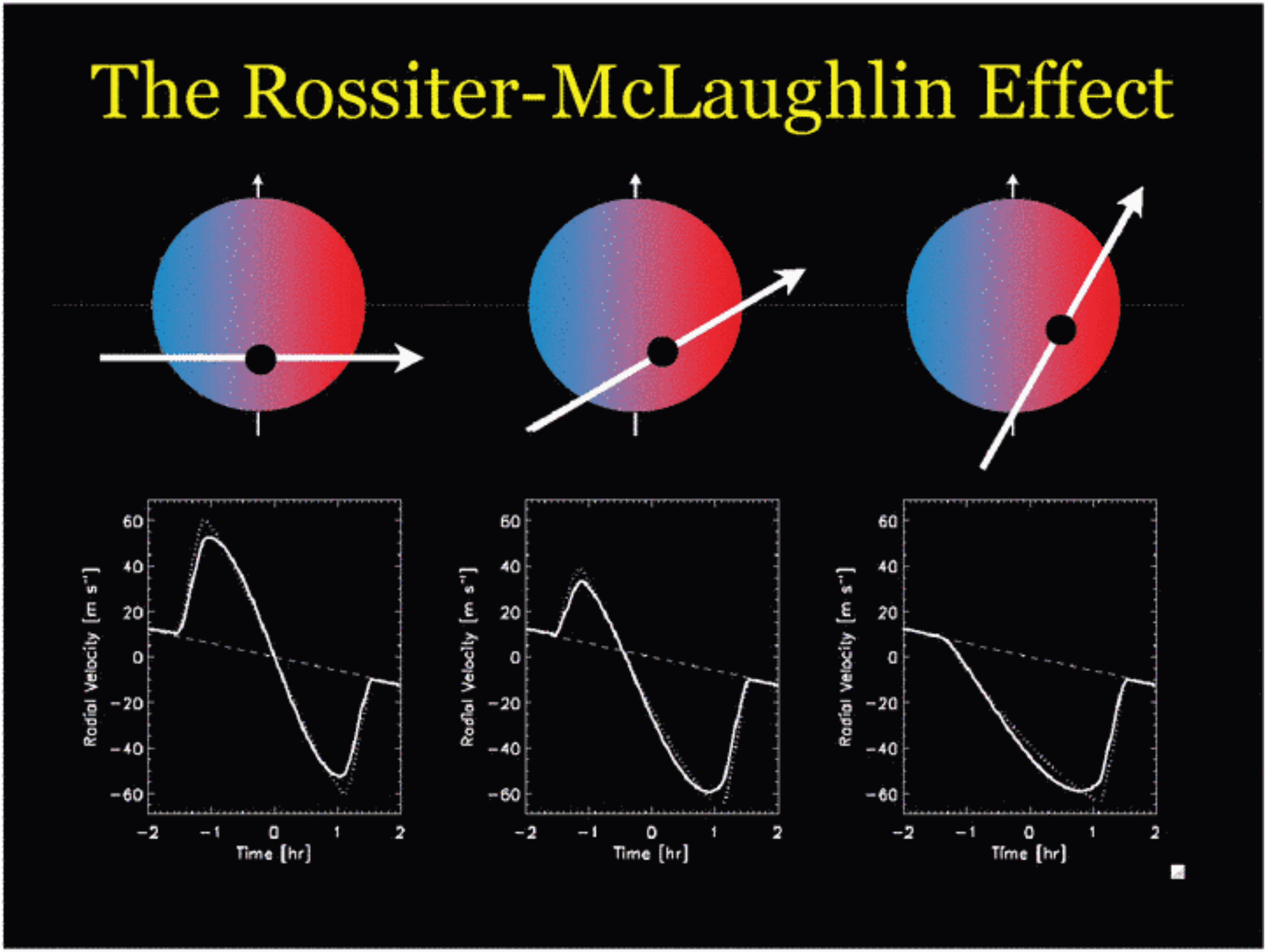


# Solar System

All major planets lie in the same plane (within 2 deg), which is  inclined to the Sun's equator by 7 deg.




exoplanets.org | 1/22/2022
Spin-Orbit Misalignment [Degrees]
100
0
-100
1
10
100
Orbital Period [Days]
Msin(i) [Jupiter Mass]
0
5
10
15
20



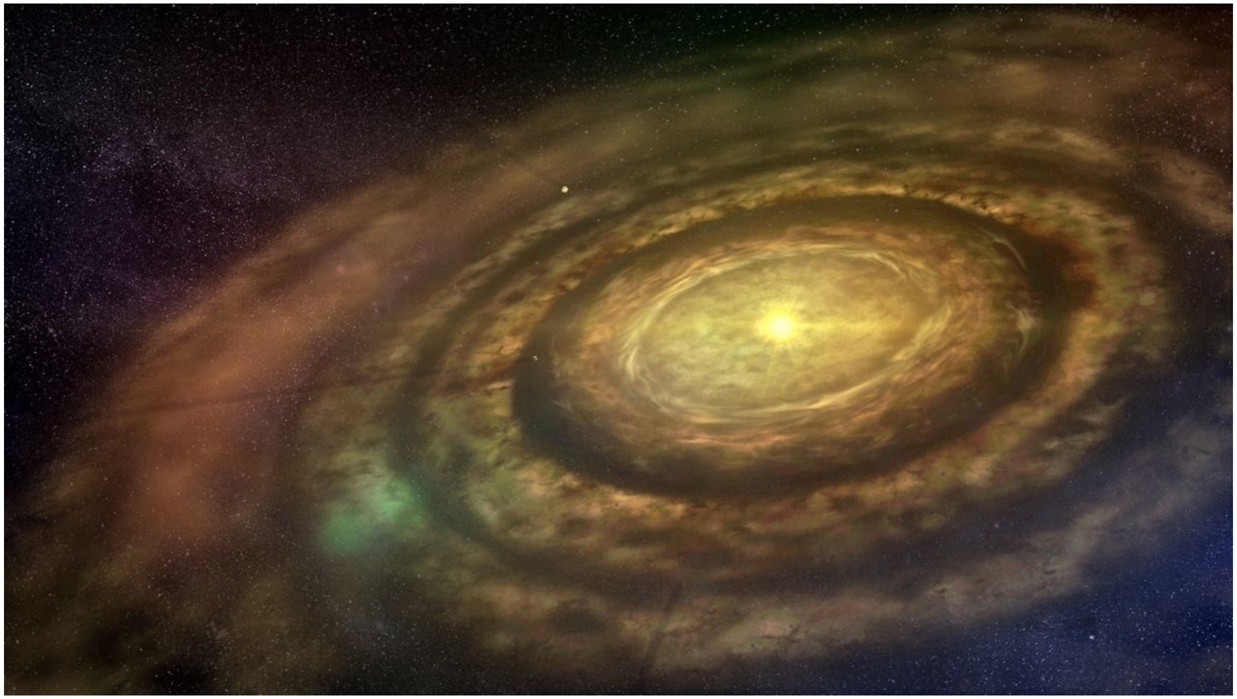

Slide from Josh Winn

# Hot Jupiter Formation

**Formation in Protoplanetary Disks (disk migration vs in-situ)**

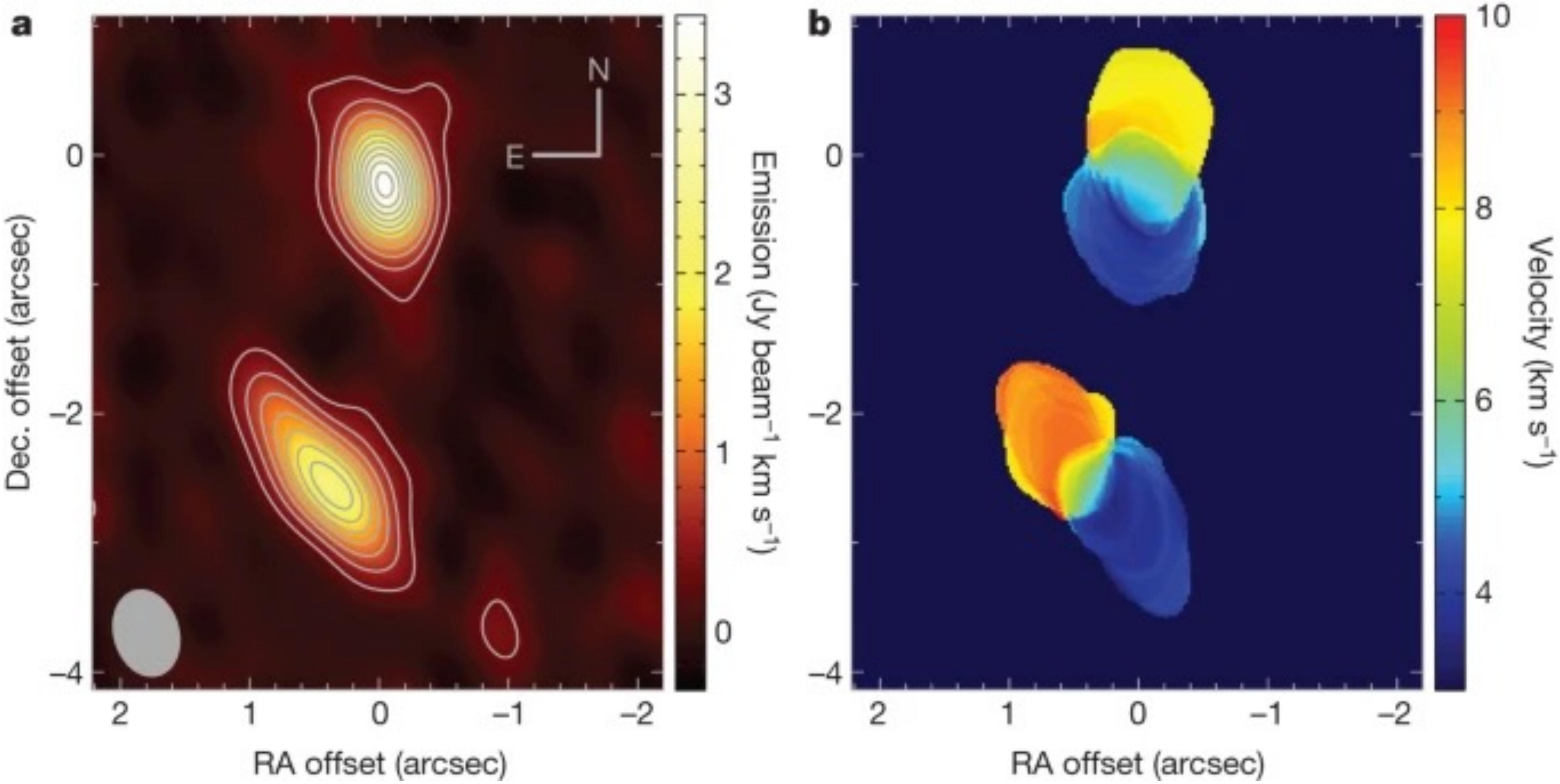
– Young proto-HJ candidates observed (e.g. CI Tau)

-- WASP-47b (HJ with small neighbors)



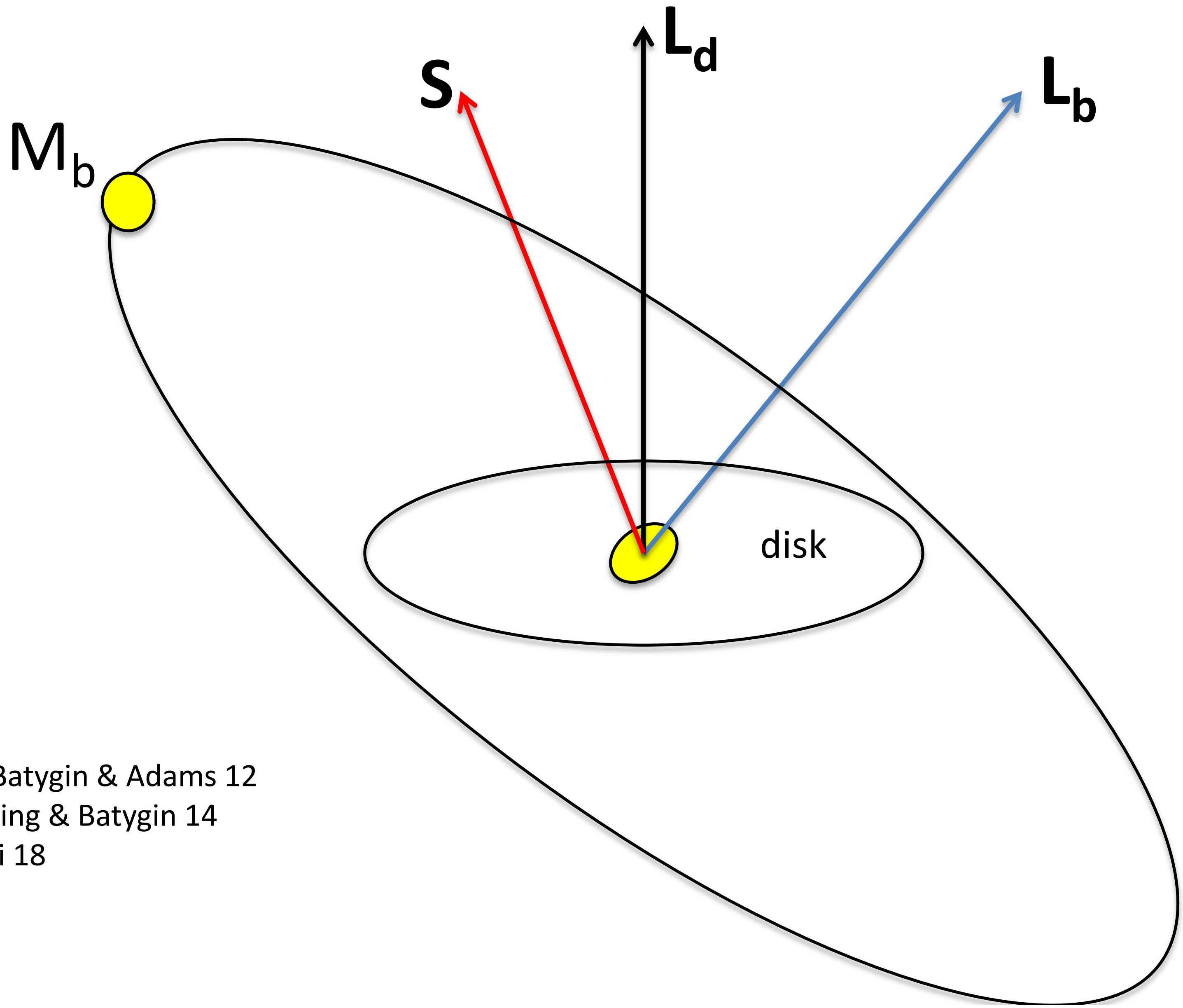
Can misalignment (stellar spin vs orbit) be produced?

# Misaligned protoplanetary disks in a binary system HK Tauri

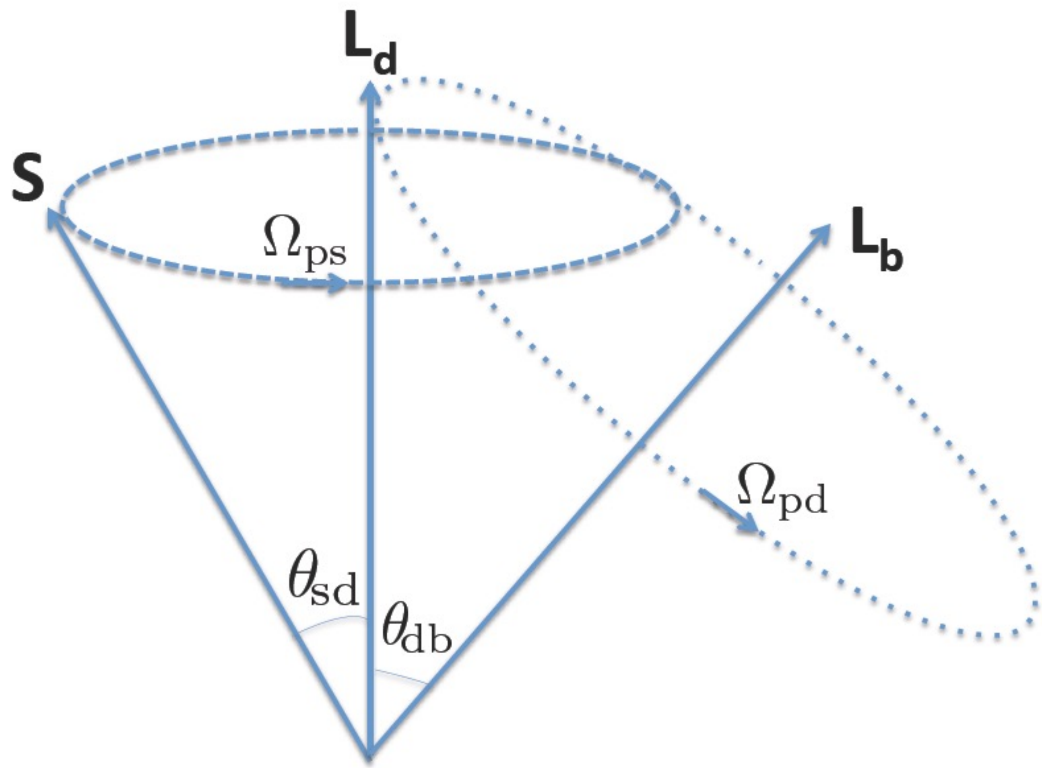



Jensen & Akeson 2014

# Star-Disk-Binary Interactions




Batygin 12, Batygin & Adams 12
Lai 14, Spalding & Batygin 14
Zanazzi & Lai 18




Disk makes stellar spin precess

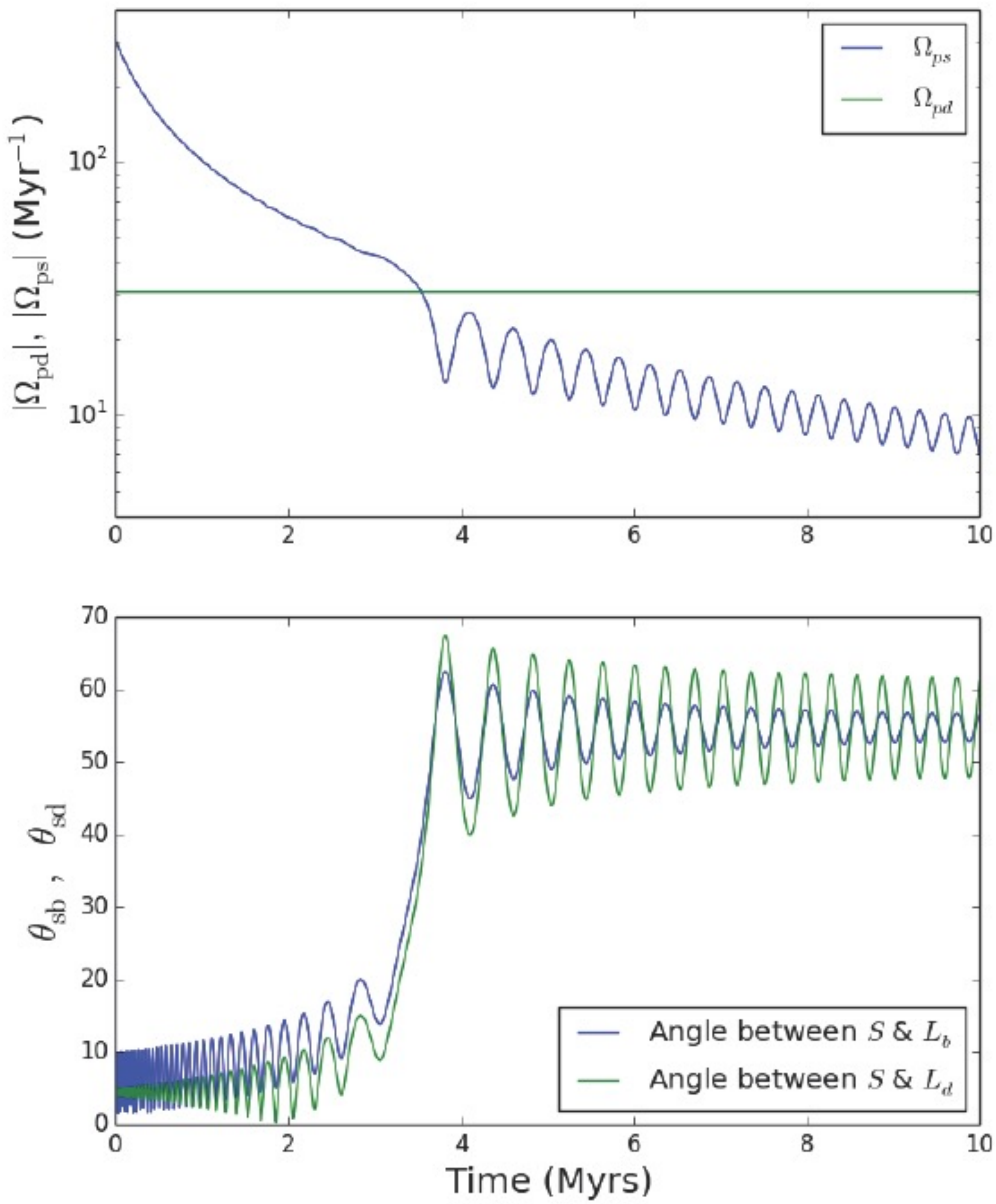
$$\Omega_{\rm ps} \simeq -5\times10^{-5}\left(\frac{M_d}{0.1M_\star}\right)\left(\frac{\bar{\Omega}_\star}{0.1}\right)\left(\frac{r_{\rm in}}{4R_\star}\right)^{-2}\left(\frac{r_{\rm out}}{50\,{\rm AU}}\right)^{-1} \times\cos\theta_{\rm sd}\left(\frac{2\pi}{\rm yr}\right)$$

Companion makes the disk precess:

$$\Omega_{\rm pd} \simeq -5\times10^{-6}\left(\frac{M_b}{M_\star}\right)\left(\frac{r_{\rm out}}{50\,{\rm AU}}\right)^{3/2}\left(\frac{a_b}{300\,{\rm AU}}\right)^{-3} \times\cos\theta_{\rm db}\left(\frac{2\pi}{\rm yr}\right)$$

Initially $|\Omega_{\rm ps}| \gg |\Omega_{\rm pd}|$

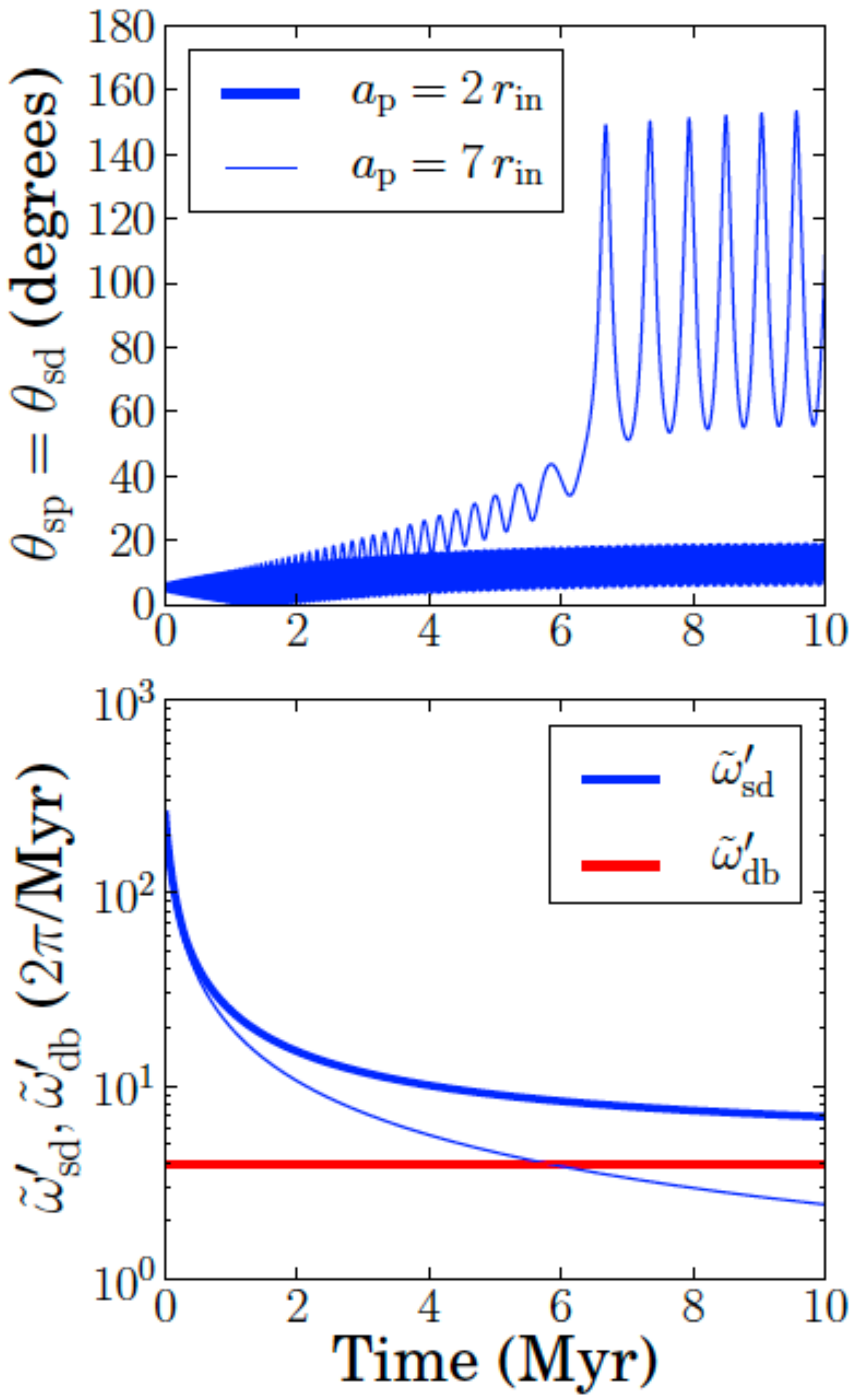
$M_d$ decreases over ~1 Myr




Resonance

produce misaligned stellar spin and disk

**Does it produce misaligned hot Jupiter?**




Can misaligned HJ be produced?

**Not necessary.**

If HJ formed (reached its close-in location) while disk is still around
➔ S strongly is coupled to the disk/HJ orbit, no resonance occurs
➔ Cannot generate large stellar obliquity.

Zanazzi & DL 2018

# Hot Jupiter Formation

## Formation in Protoplanetary Disks (disk migration vs in-situ)

– Young proto-HJ candidates observed (e.g. CI Tau)

-- WASP-47b (HJ with small neighbors)

**– Can misalignment (stellar spin vs orbit) be produced? Not easy…**

## High-Eccentricity Migration

(e.g. Eggleton+01; Wu & Murray 03; Fabrycky & Tremaine 07; Nagasawa+08; Wu & Lithwick 11; Correia et al.11; Beauge & Nesvorny 12; Naoz+12; Storch et et al.14; Petrovich 15a,b; Anderson+16; Munoz & Lai+16; Wu 18; Vick & Lai+19; Teyssandier, Lai+19… )

# Hot Jupiter formation: High-e Migration

- Planet forms at ~ a few AU
- Interaction with companion (other star or planets) pumps planet into high-e orbit (Lidov-Kozai, "secular chaos", etc).
- Tidal dissipation in planet during high-e phases causes orbital decay

➔ Combined effects can result in planets in ~ few days orbit

e.g. Eggleton+01; Wu & Murray 03; Fabrycky & Tremaine 07; Nagasawa+08; Wu & Lithwick 11; Beauge & Nesvorny 12; Naoz+12; Storch, Anderson & Lai 14; Petrovich 15a,b; Anderson, Lai +16; Munoz, Lai +16; Wu 18; Vick, Lai +19; Teyssandier, Lai+19

**Main physics uncertainties:**

Need strong tidal dissipation; Can HJ survive tidal disruption? ...

Nonlinear "Chaotic tides": Vick & Lai 2018; Wu 2018; Vick, Lai & Anderson 2019; Yu+ 2021

# Hot Jupiter formation: High-e Migration

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**How to produce misaligned stellar spin?**

As an example, I will focus on Lidov-Kozai driven migration

# Lidov-Kozai Effect

Can perturbation from the Moon make Earth's satellites fall?

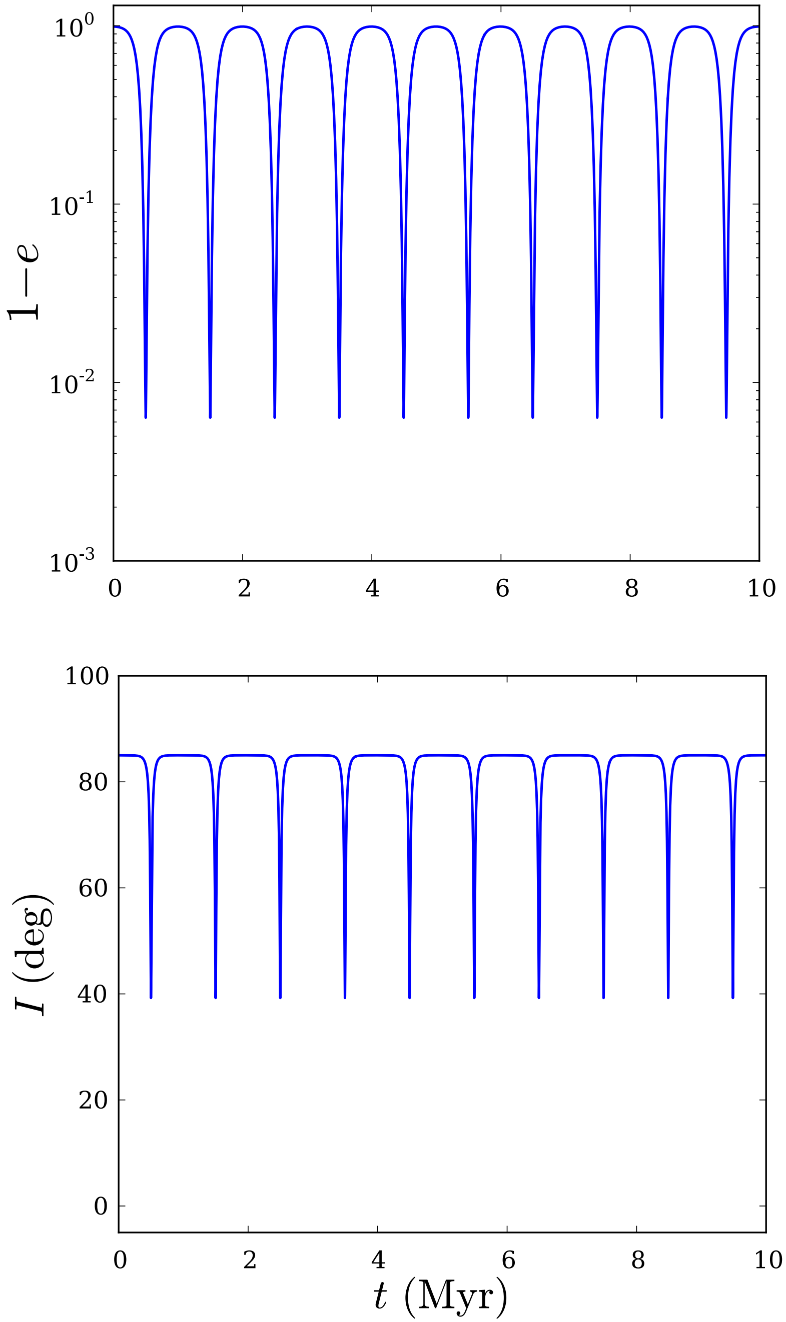
Planet. Space Sci., 1962, Vol. 9, pp. 719 to 759. Pergamon Press Ltd. Printed in Northern Ireland

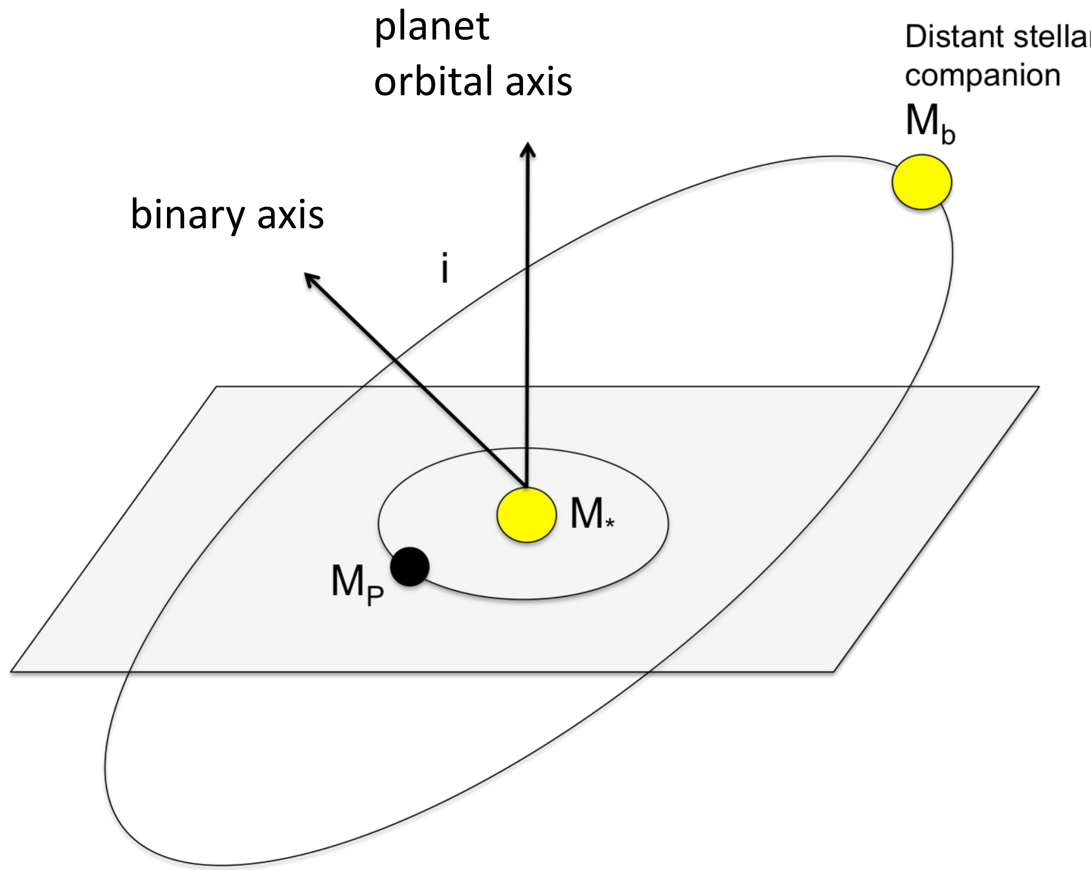
THE EVOLUTION OF ORBITS OF ARTIFICIAL SATELLITES OF PLANETS UNDER THE ACTION OF GRAVITATIONAL PERTURBATIONS OF EXTERNAL BODIES

M. L. LIDOV

Translated by H. F. Cleaves from *Iskusstvennye Sputniki Zemli*, No. 8, p. 5, 1961.

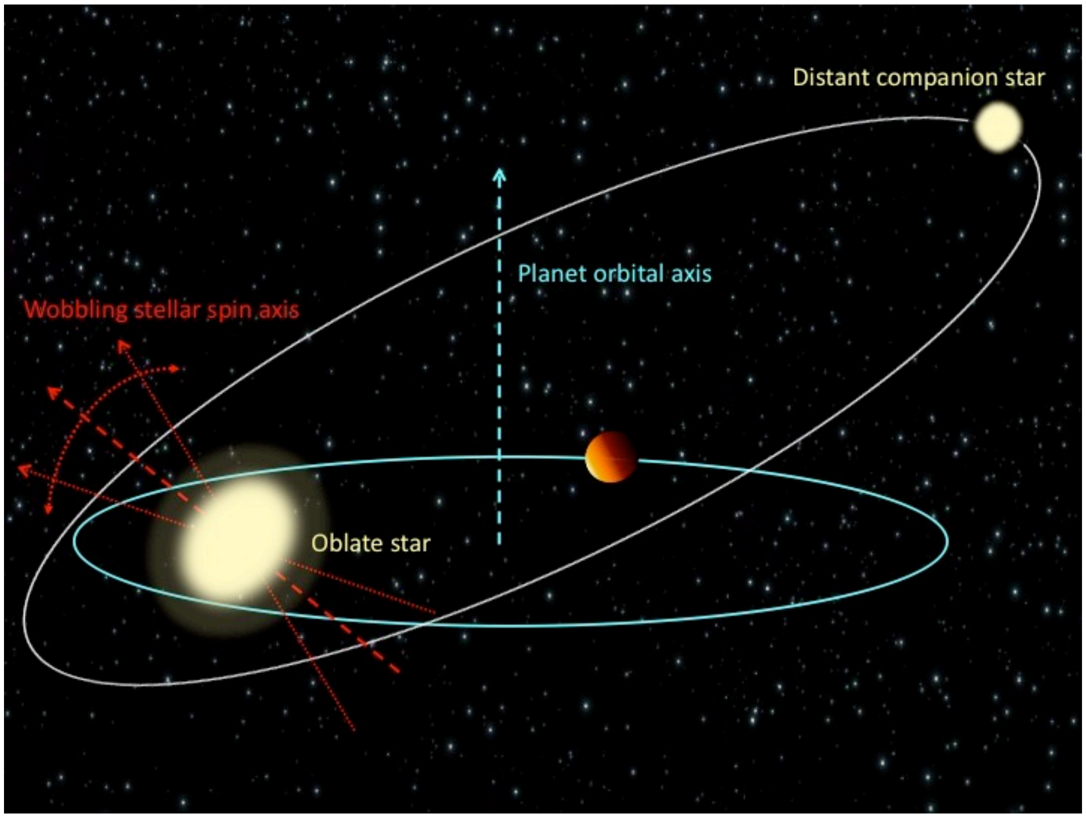
# Lidov-Kozai Effect




- Eccentricity and inclination oscillations induced if i > 40 degrees.
- If i large (85-90 degrees), get extremely large eccentricities (e > 0.99)

# What happens to stellar spin axis as the planet undergoes Lidov-Kozai Oscillations ?



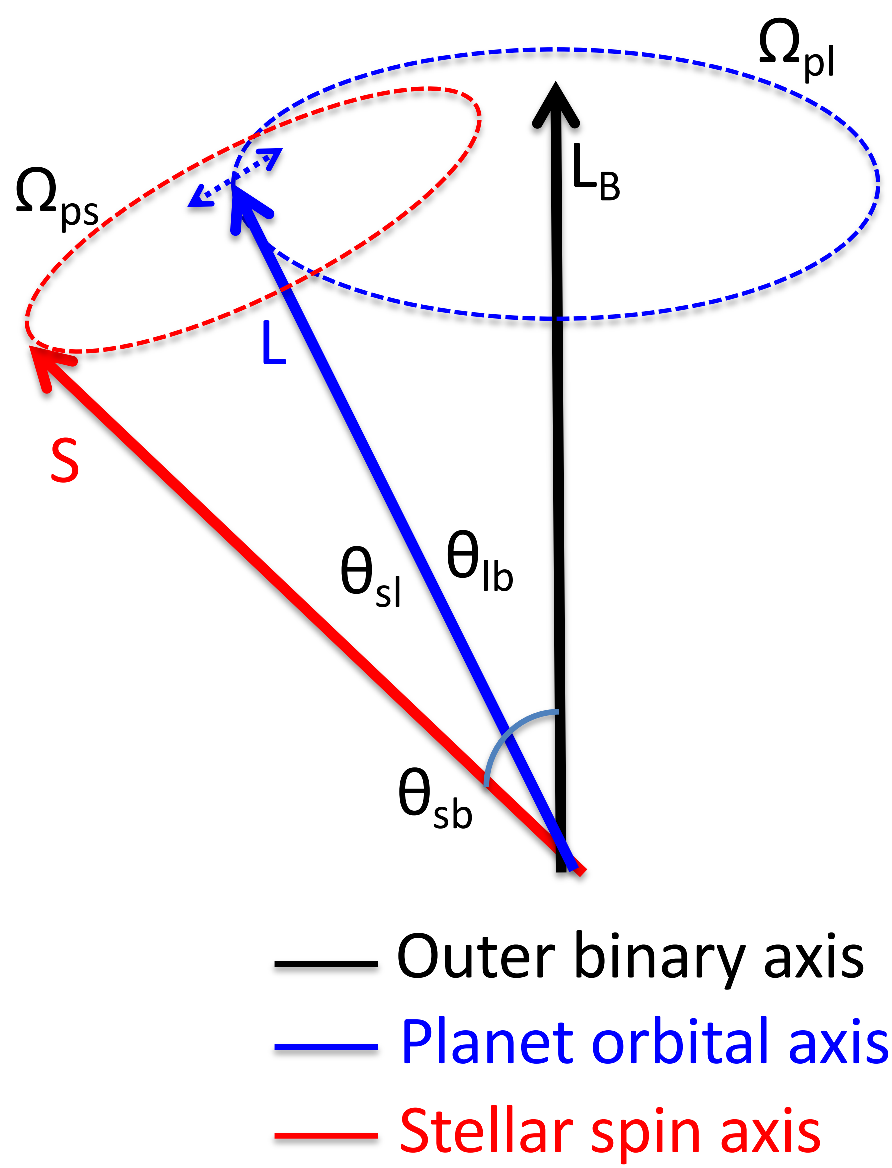

Star rotates ➔ oblate
➔ **S** precesses around **L**

$$\Omega_{\rm ps} = -\frac{3GM_p(I_3 - I_1)}{2a^3(1-e^2)^{3/2}}\frac{\cos\theta_{\rm sl}}{S}$$
$$\propto \frac{\Omega_{\rm s} M_{\rm p}}{a^3(1-e^2)^{3/2}}$$

Storch, Anderson & DL 2014, Science
Storch & DL 2015 MNRAS (Theory I)
Anderson, Storch, DL 2016 (Pop Study)
Storch, DL & Anderson2017 (Theory II)
Vick, DL & Anderson 2019

# Spin Dynamics

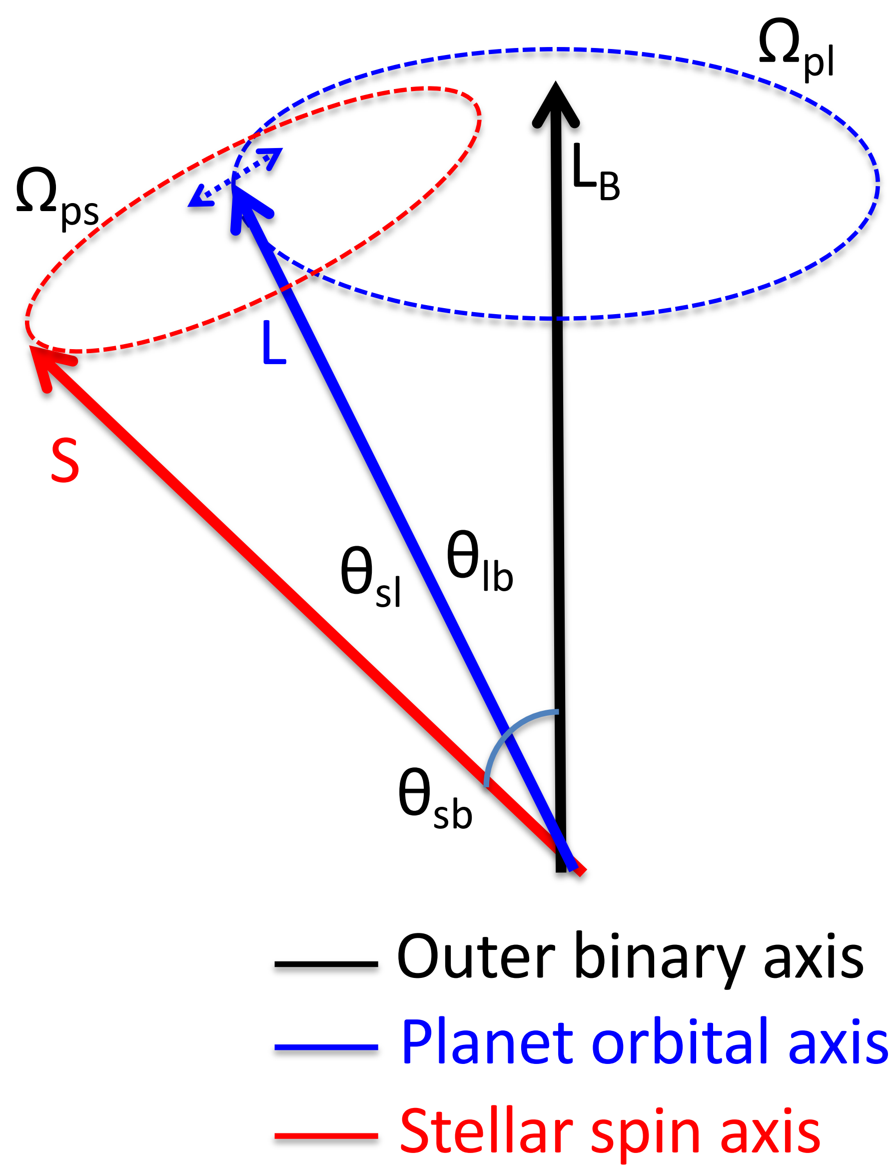
- Stellar spin axis S tries to precess around planet orbital axis L.
- But L itself is moving:
  - Nodal precession (L precesses around binary axis $L_b$)
  - Nutation (cyclic changes in inclination of L relative to $L_b$)



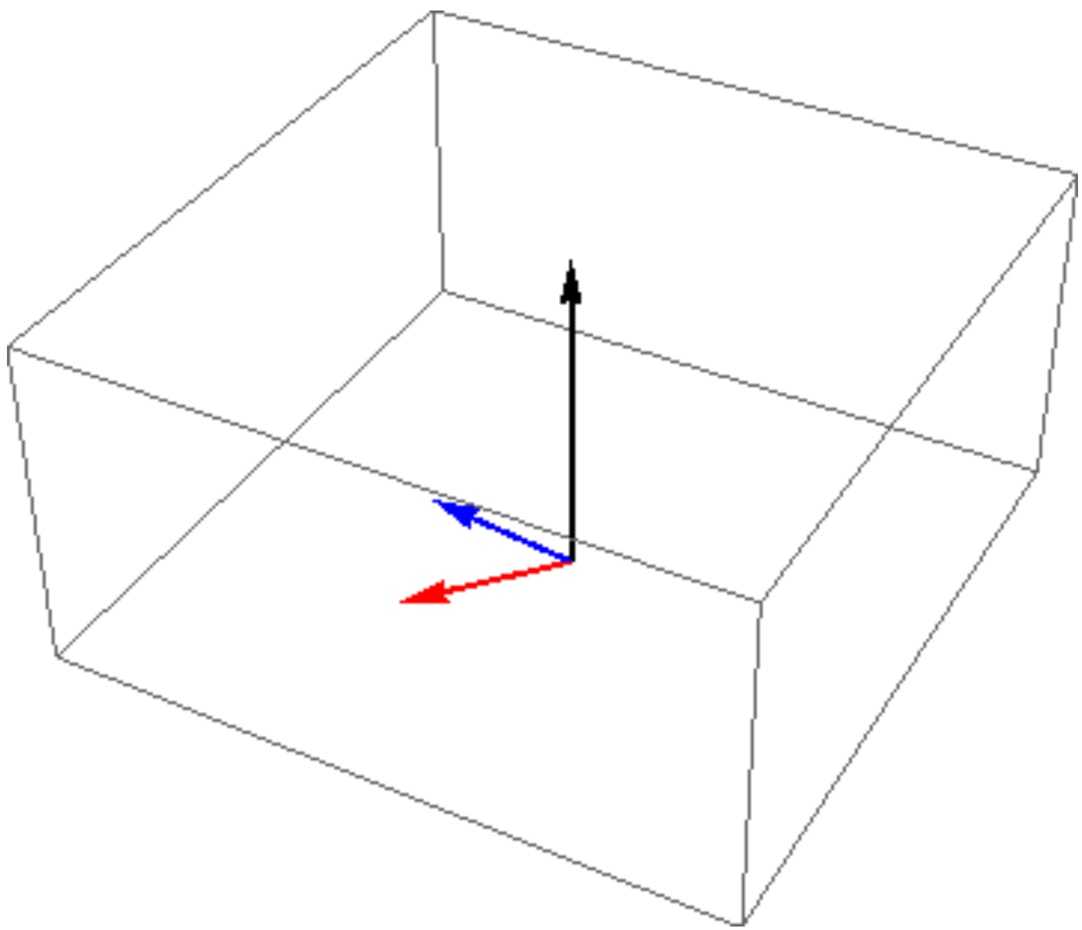
# Spin Dynamics

- **Can S keep up with L?**
- Answer depends on

$$\Omega_{\rm ps} \quad \text{vs} \quad \Omega_{\rm pl}$$



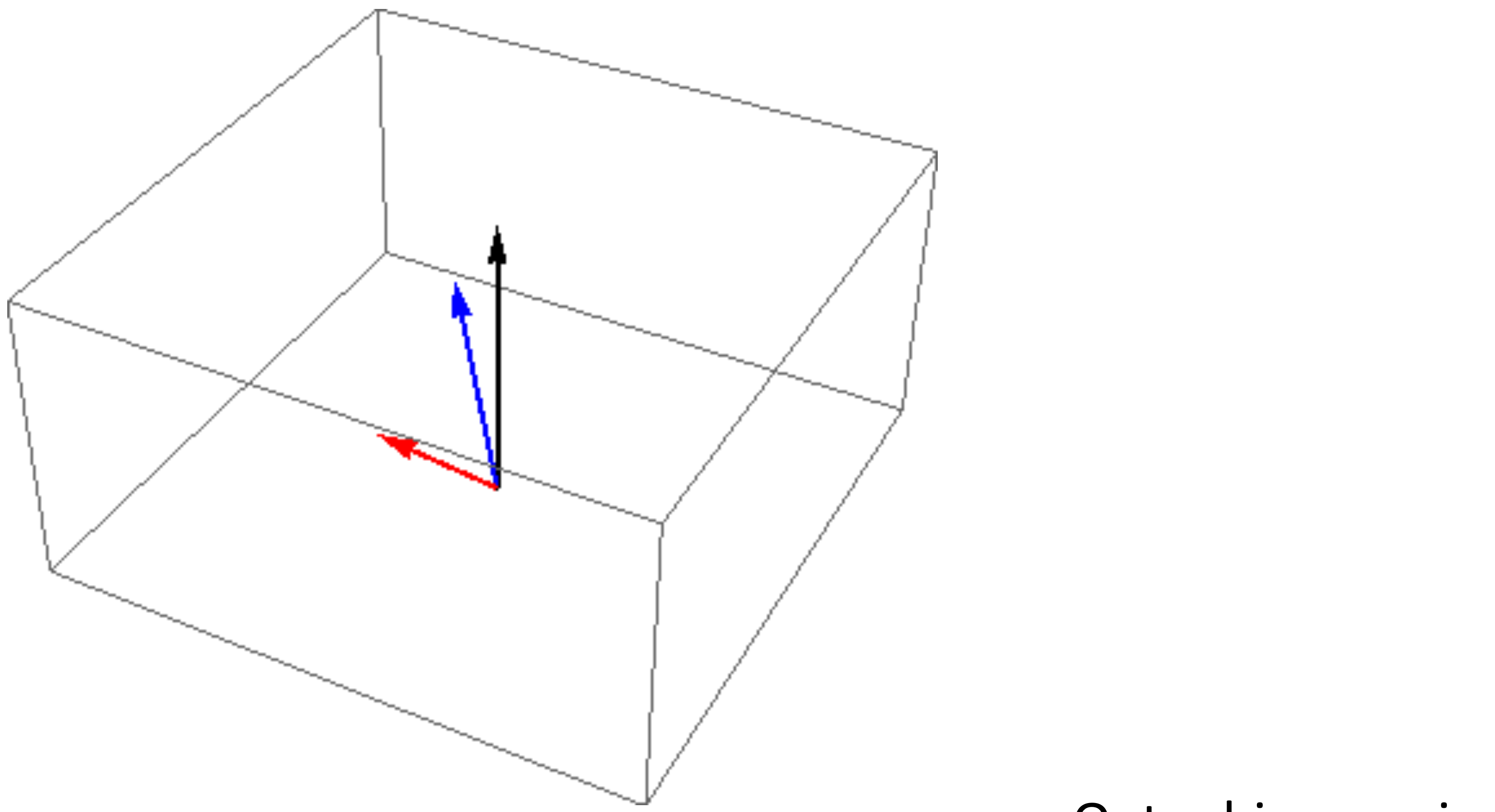
# If $|\Omega_{ps}| >> |\Omega_{pl}|$: YES ("adiabatic")



$\theta_{sl}$ = constant, i.e. initial spin-orbit misalignment is maintained for all time

Outer binary axis
Planet orbital axis
Stellar spin axis

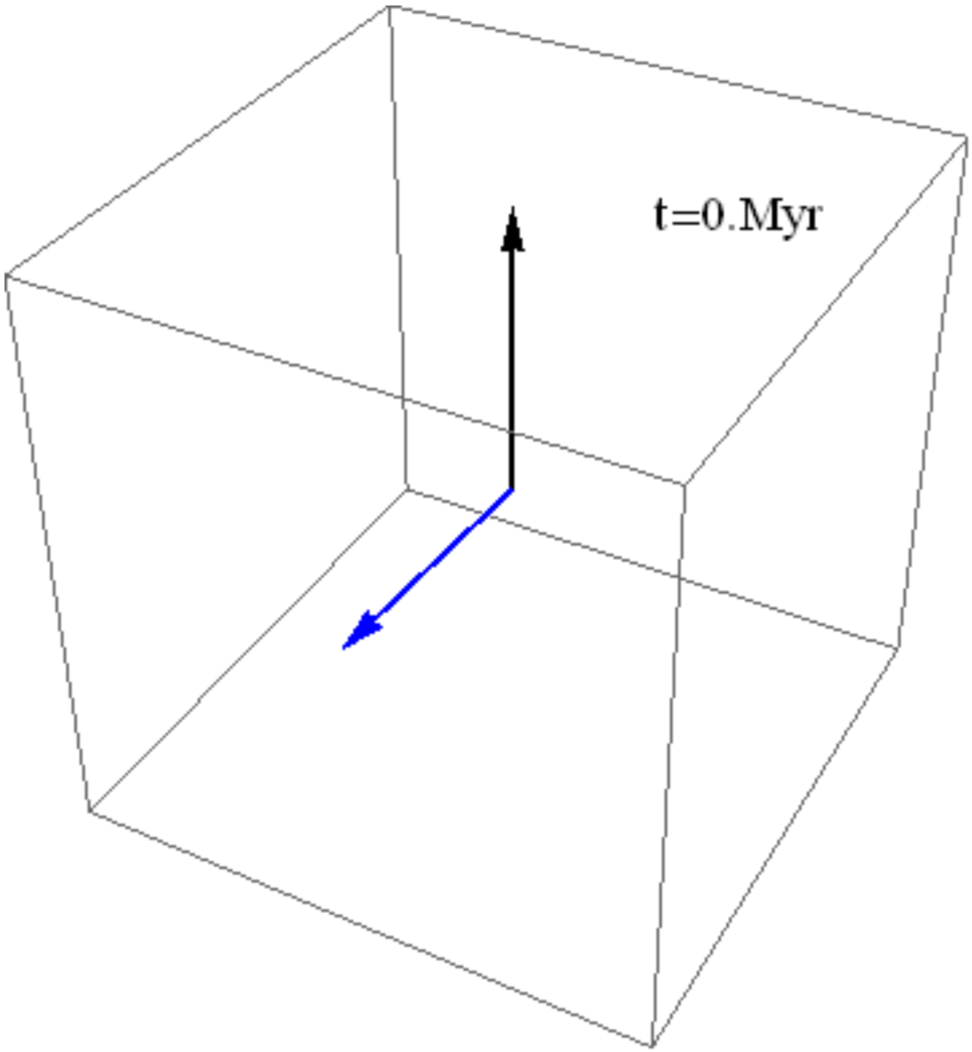
# If $|\Omega_{ps}| << |\Omega_{pl}|$: NO (“non-adiabatic”)



# If $|\Omega_{ps}| \sim |\Omega_{pl}|$: "trans-adiabatic"

To answer, need to solve orbital evolution equations together with spin precession equation….

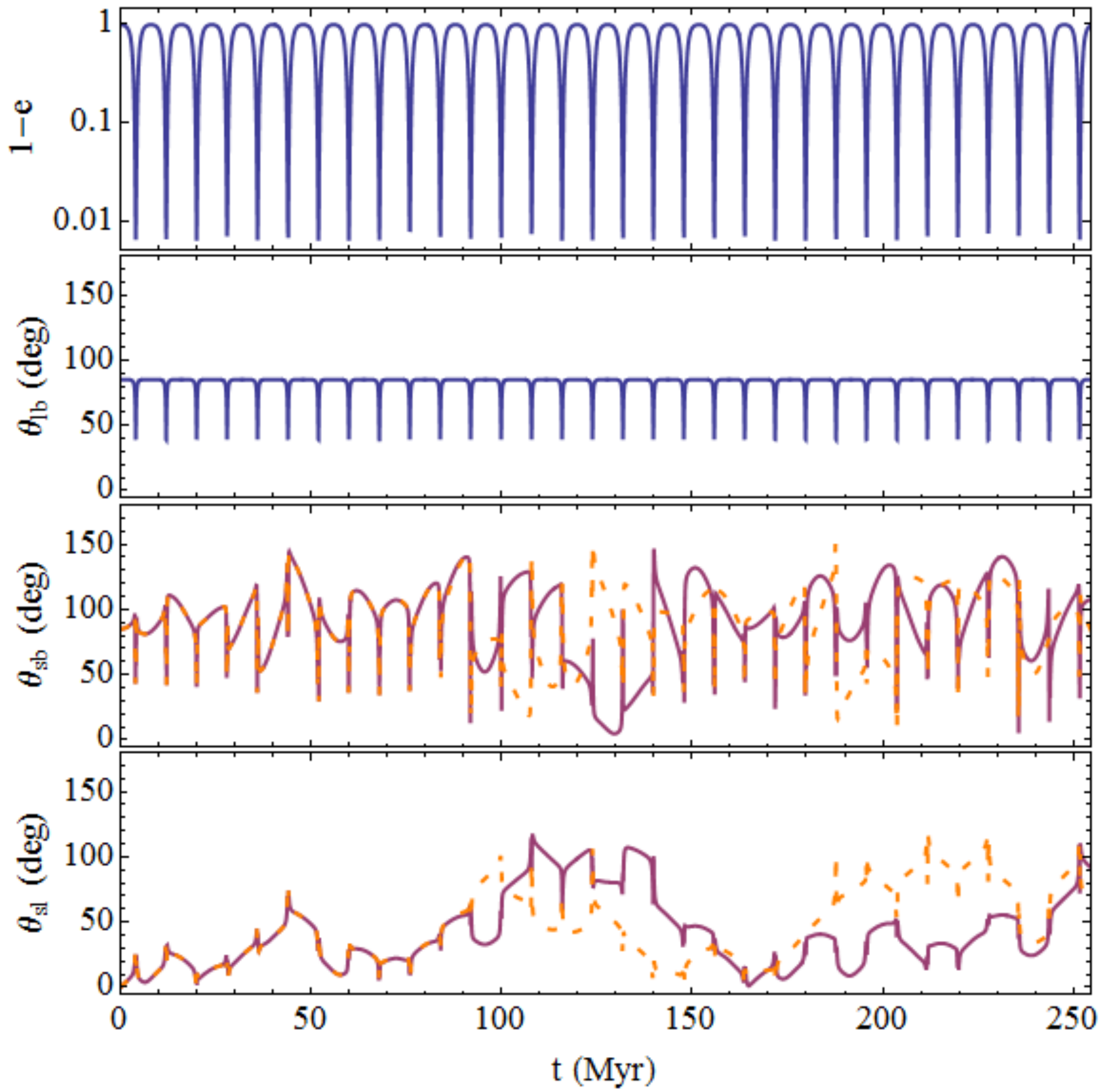
If $|\Omega_{ps}| \sim |\Omega_{pl}|$: "trans-adiabatic"

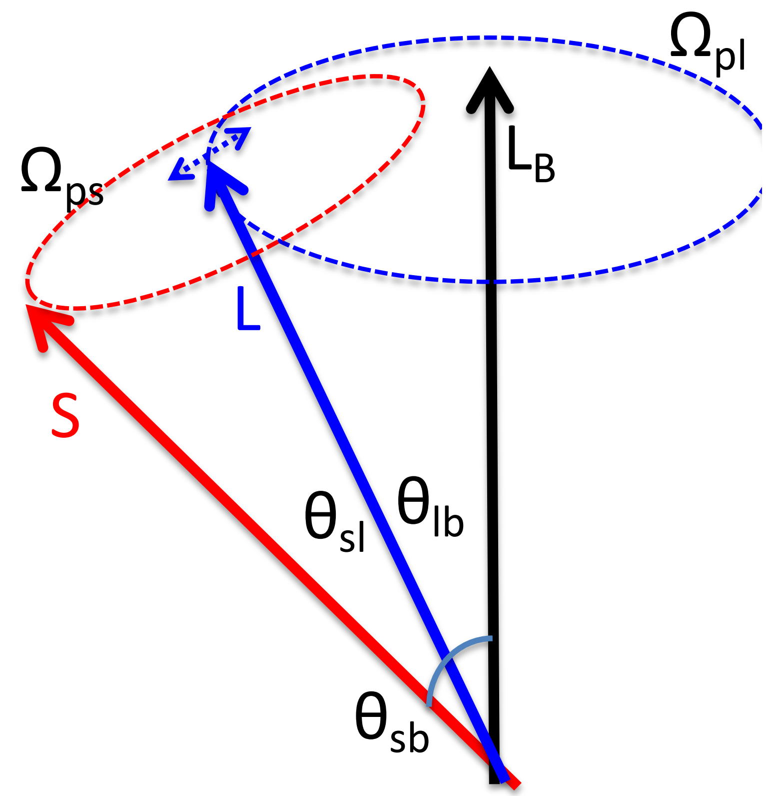



Q: Is it really chaotic?

— Outer binary axis
— Planet orbital axis
— Stellar spin axis

# If $|\Omega_{ps}| \sim |\Omega_{pl}|$: “trans-adiabatic”



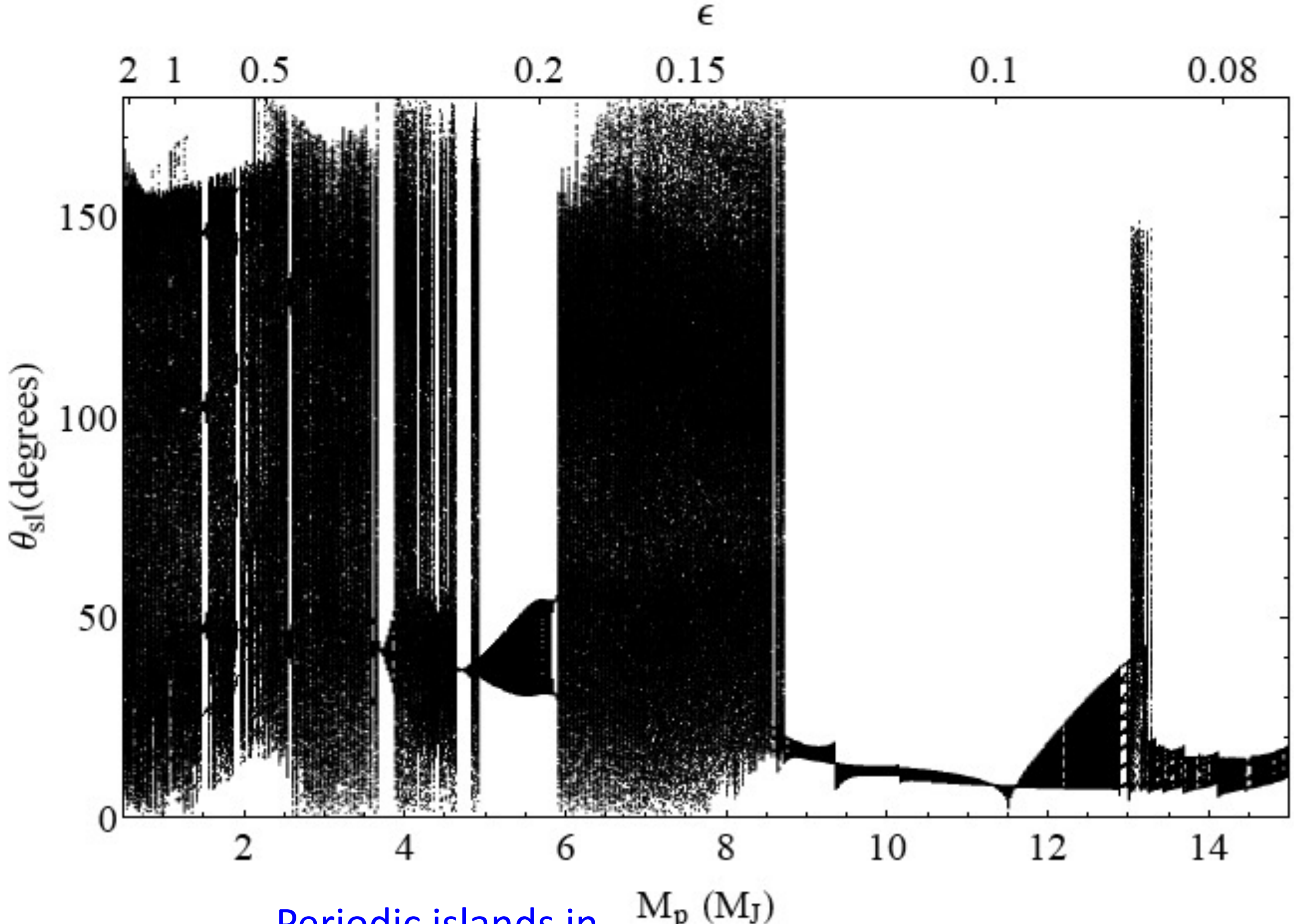




Lyapunov time ~ 6 Myrs

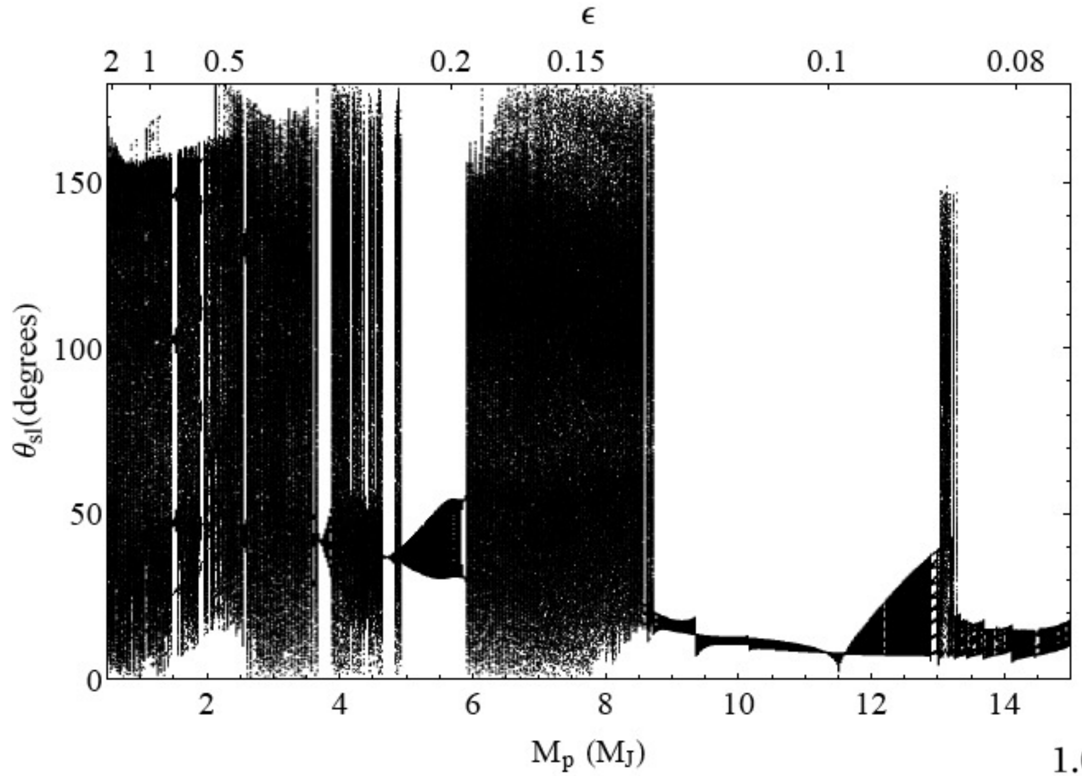
$\Omega_{ps}$ & $\Omega_{pl}$ are strong functions of eccentricity (and time)

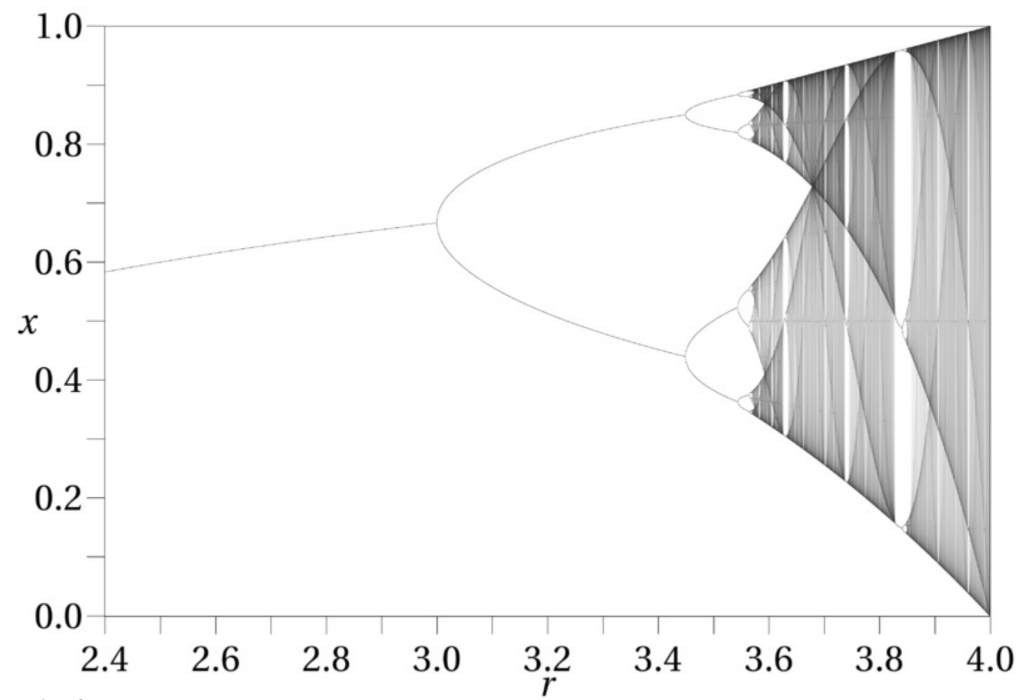
$$\epsilon = \frac{\Omega_{\mathrm{pl0}}}{\Omega_{\mathrm{ps0}}} \propto M_p^{-1}$$

Spin-orbit angle recorded at eccentricity maxima




Periodic islands in the ocean of chaos



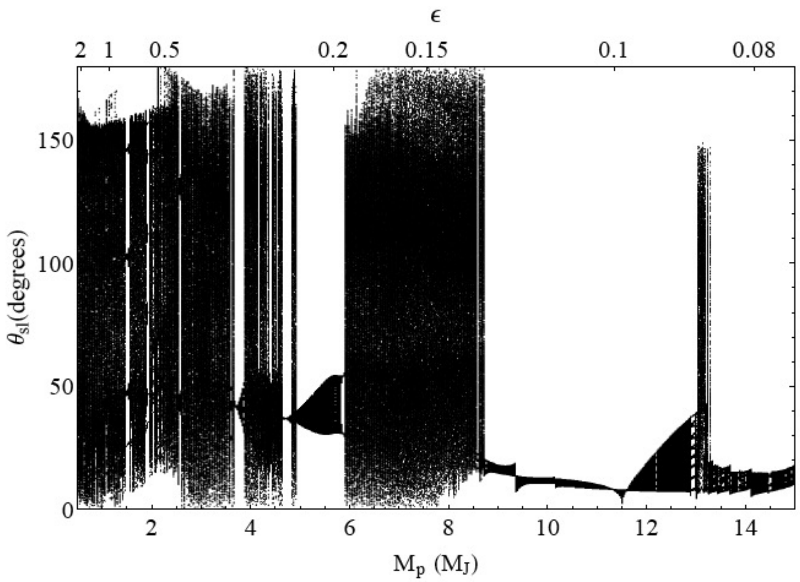




Logistic Map:

$$x_{n+1} = rx_n(1 - x_n)$$

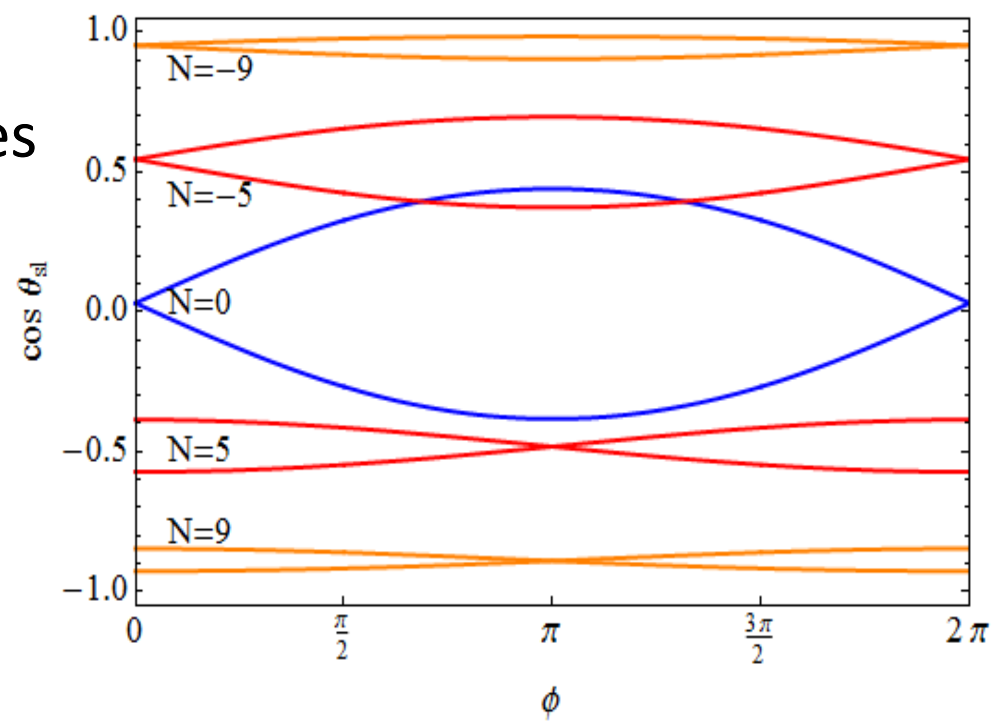
R.May (1976): Discrete time population model

# Theory of Spin Chaos



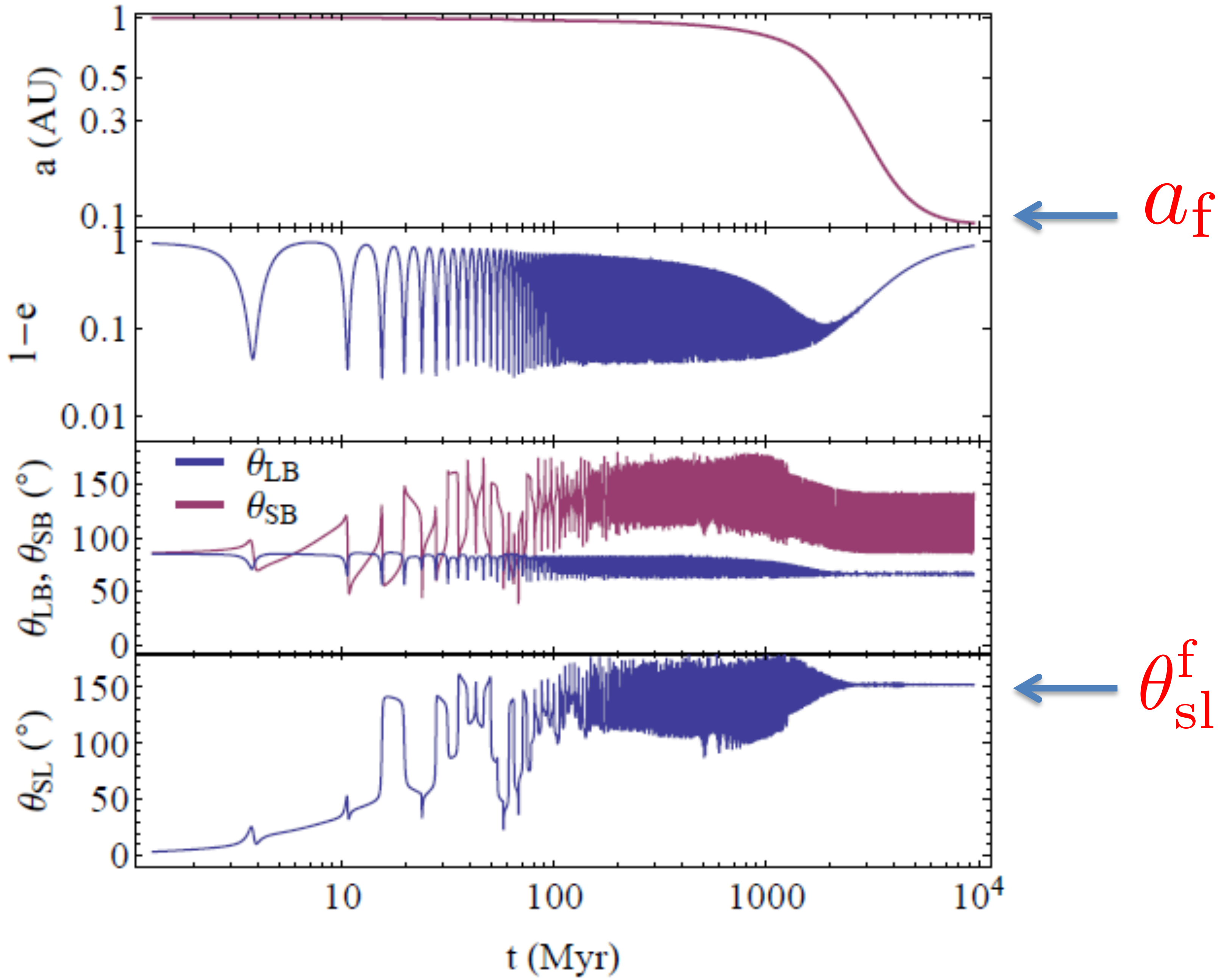

In Hamiltonian system, Chaos arises from **overlapping resonances** (Chirikov criterion; 1979)

Storch, Lai 2015,2017



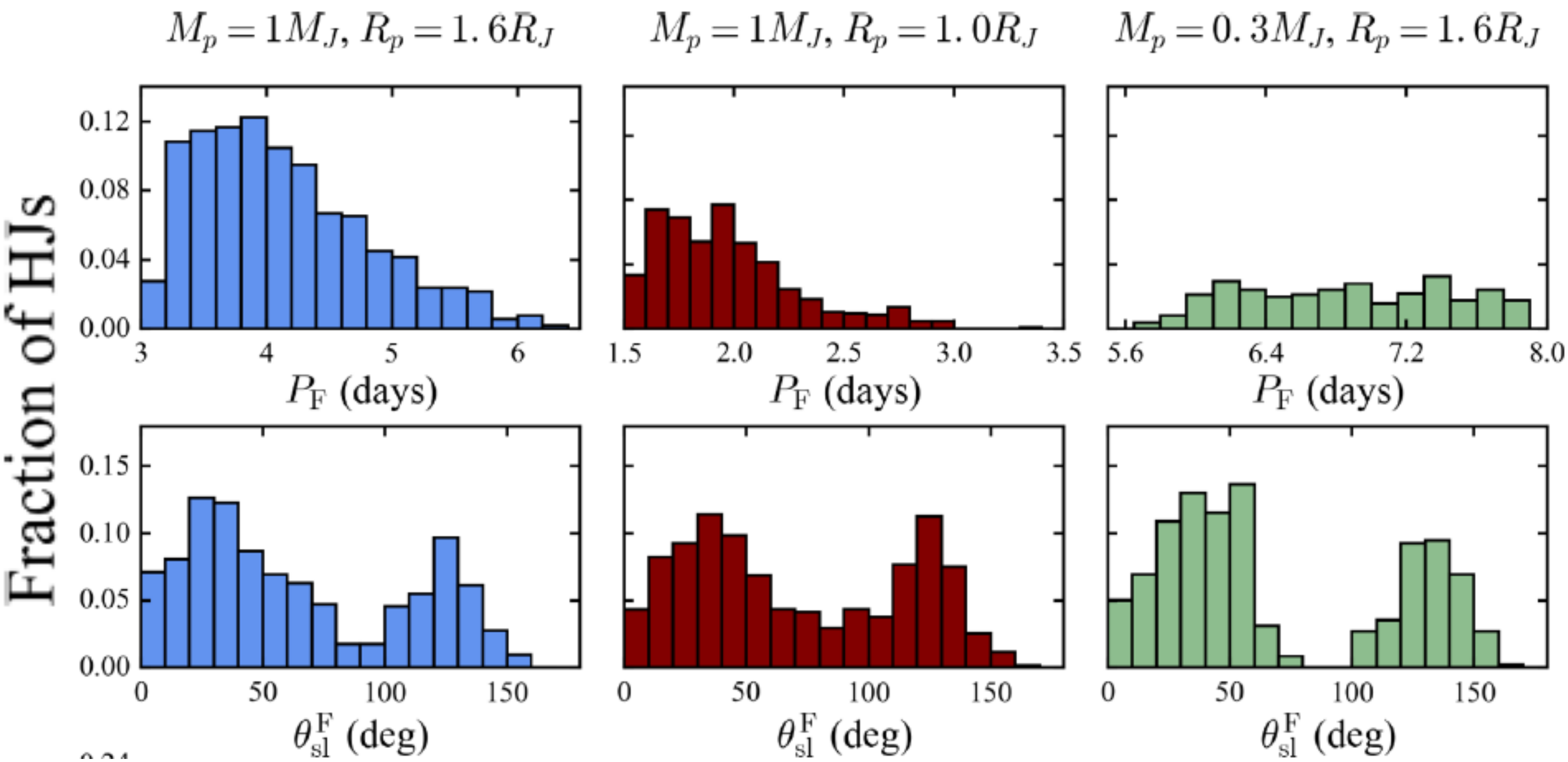
# Add Tidal Dissipation….

# Lidov-Kozai + Tidal Dissipation



# Formation of Hot Jupiters via Lidov-Kozai High-e Migration

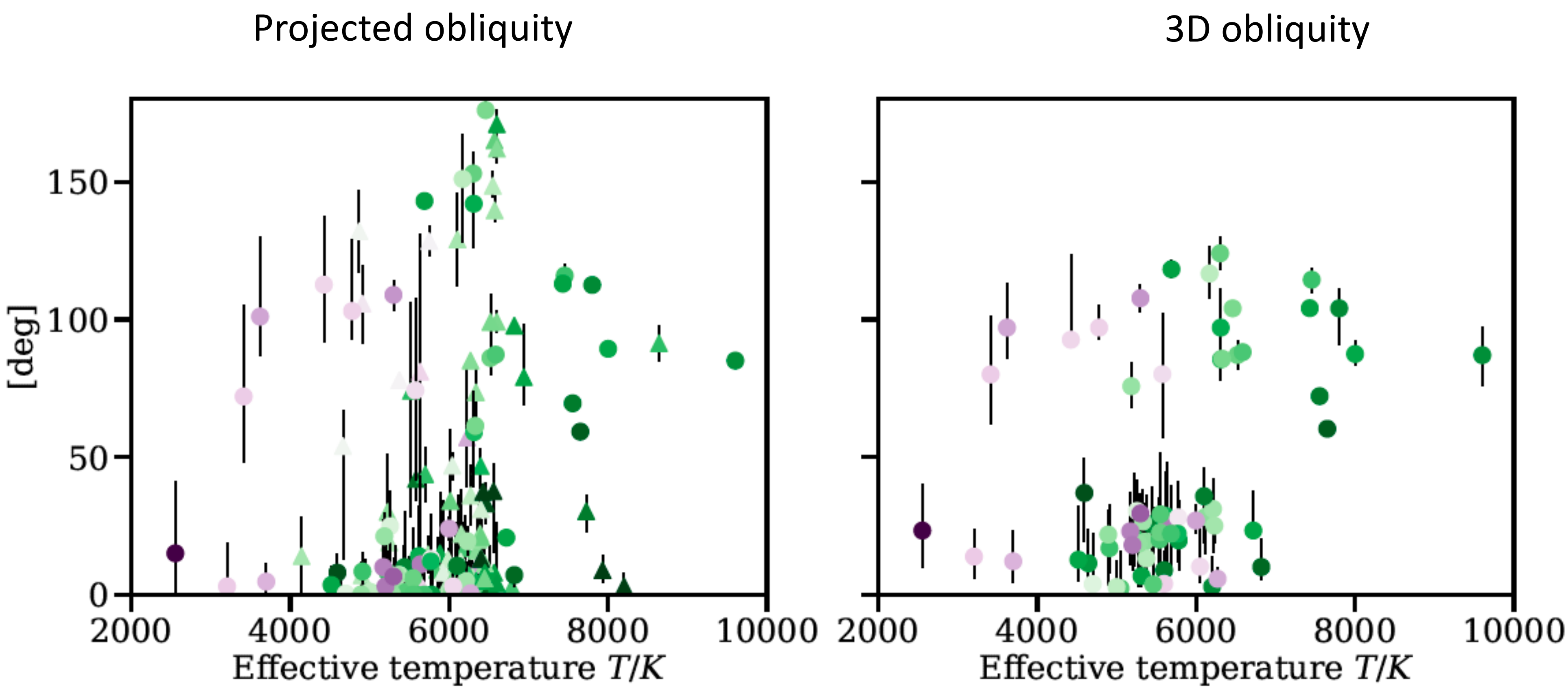
Vick, DL & Anderson 2019




Produce a bi-modal distribution of stellar obliquities (bifurcation phenomenon)

Seen also in some previous works…

# Observational data



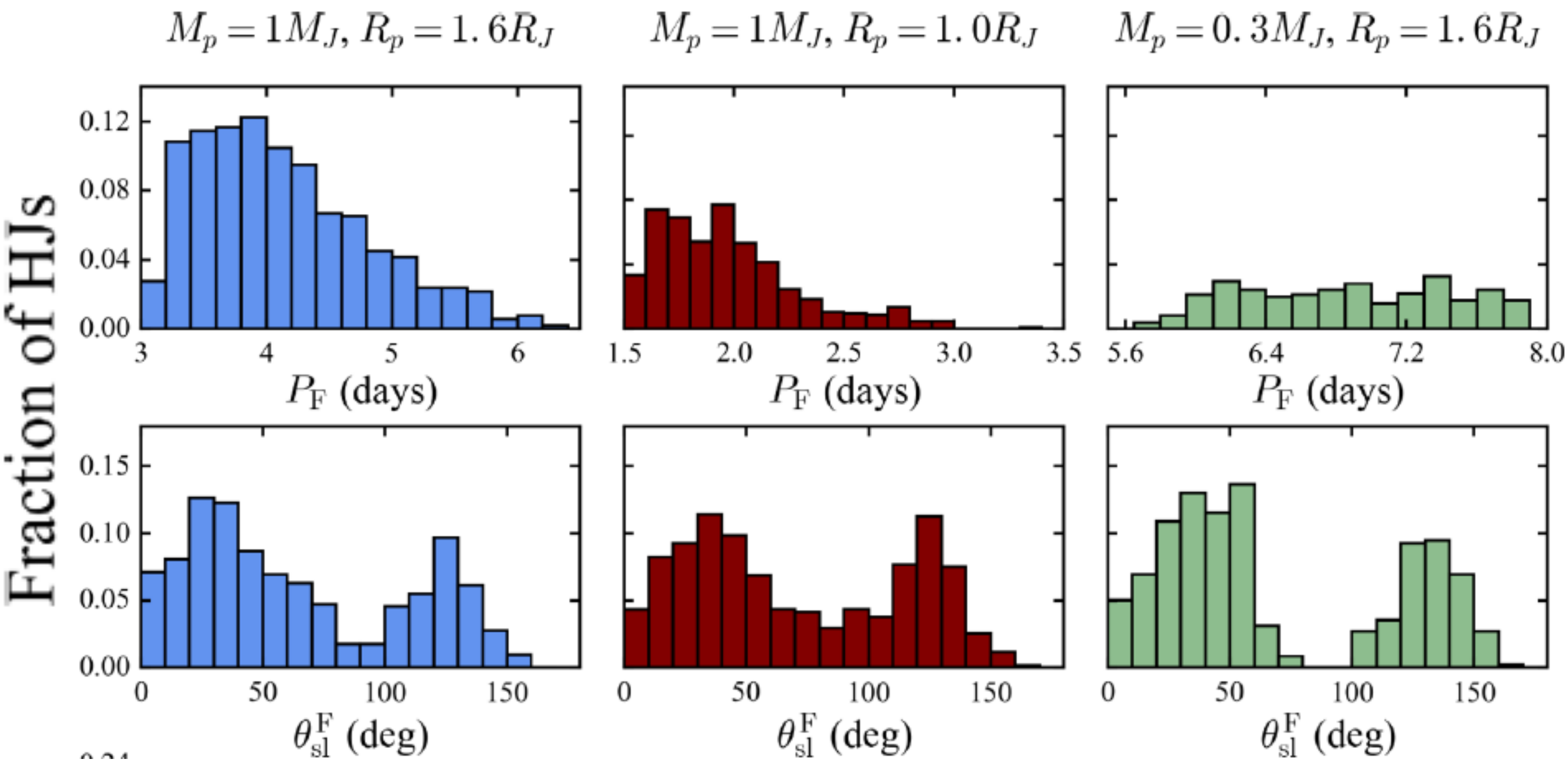

Siegel, Winn & Albrecht 2023

Two populations: aligned systems

+ largely isotropic systems

# Formation of Hot Jupiters via Lidov-Kozai High-e Migration

Vick, DL & Anderson 2019

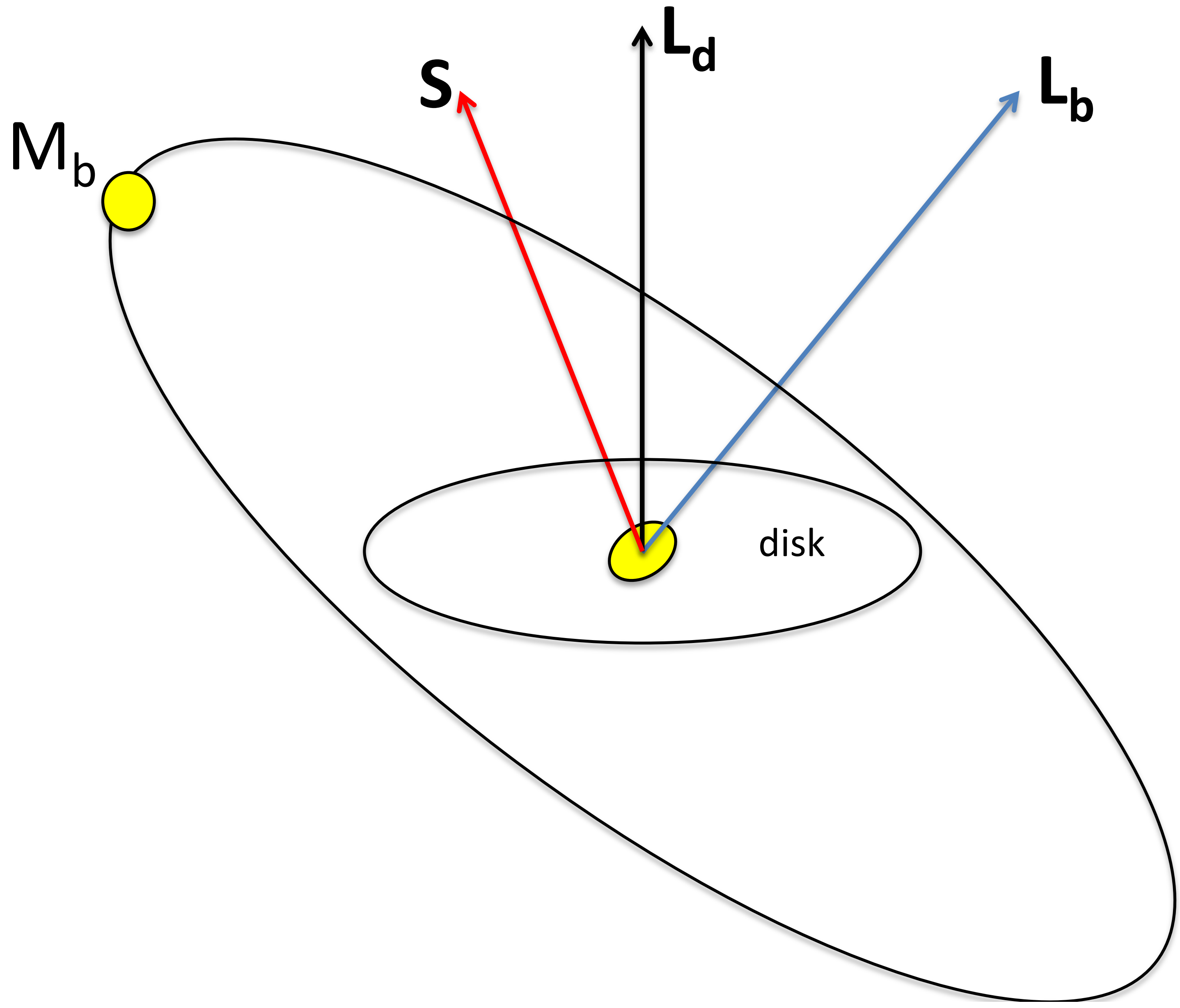



Produce a bi-modal distribution of stellar obliquities (bifurcation)

This assumes that the stellar spin axis is aligned with cold Jupiter orbital axis prior to high-e migration

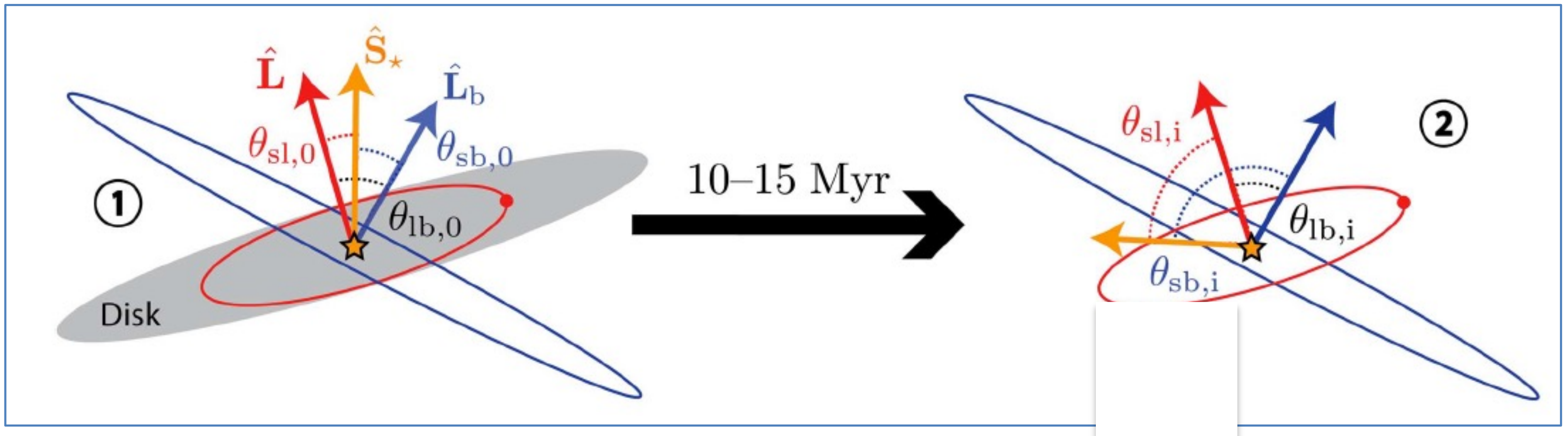
Before high-e migration takes place, we have:

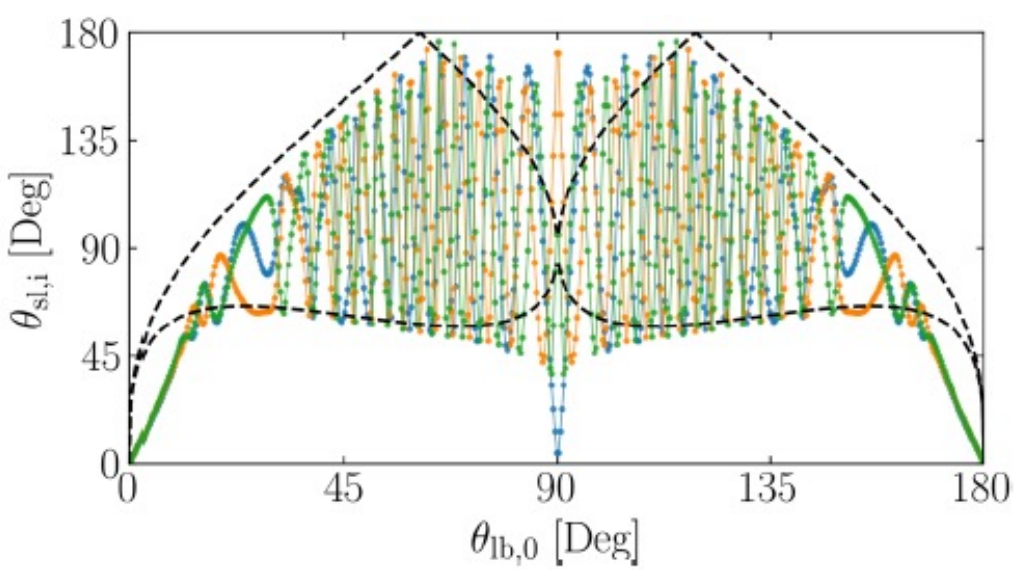
# Star-Disk-Binary Interactions



# Before high-e migration takes place, we have:

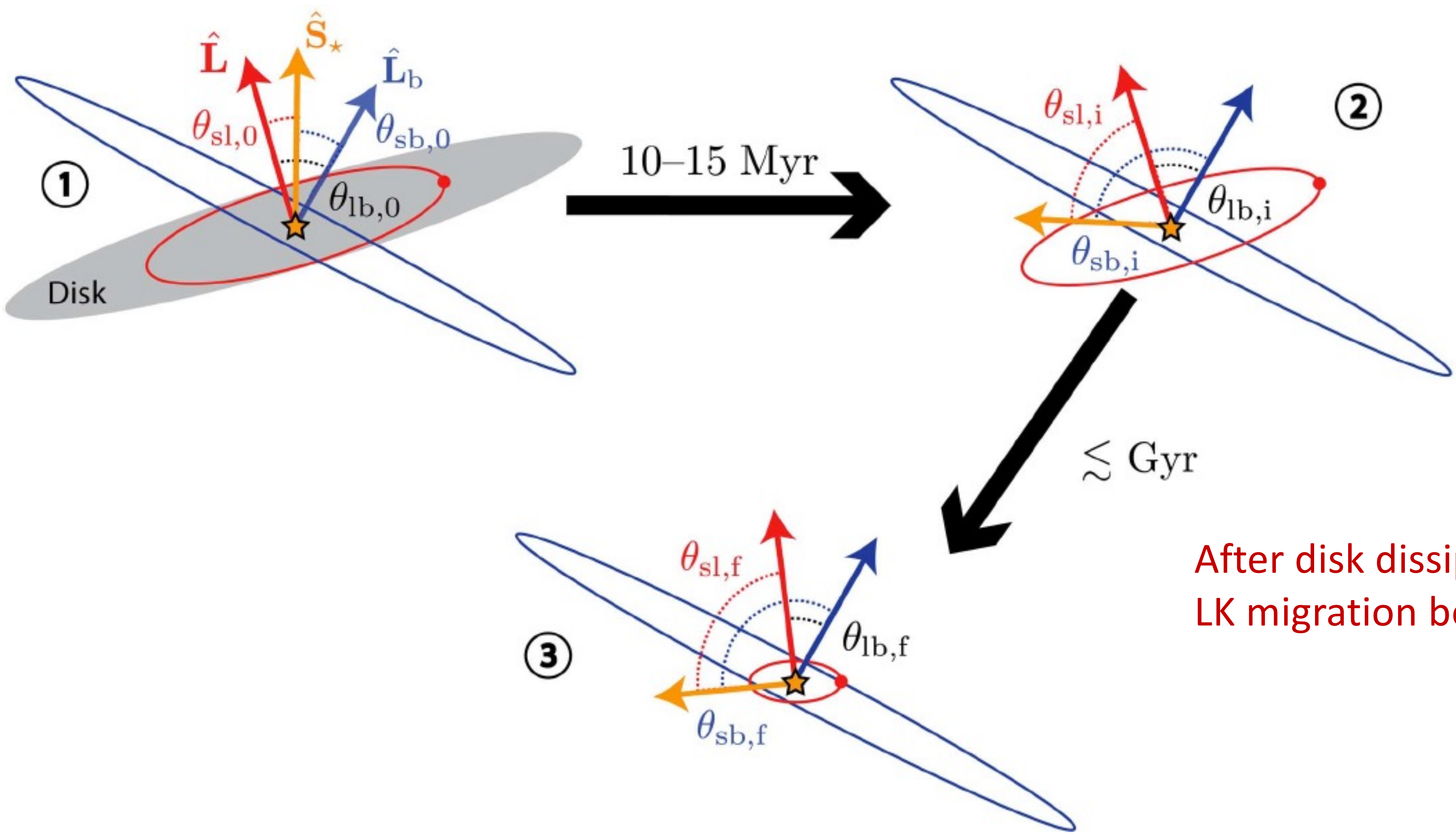
Cold Jupiter formed/embedded in disk (PPD):  L
Binary companion $L_b$
Stellar spin $S_*$



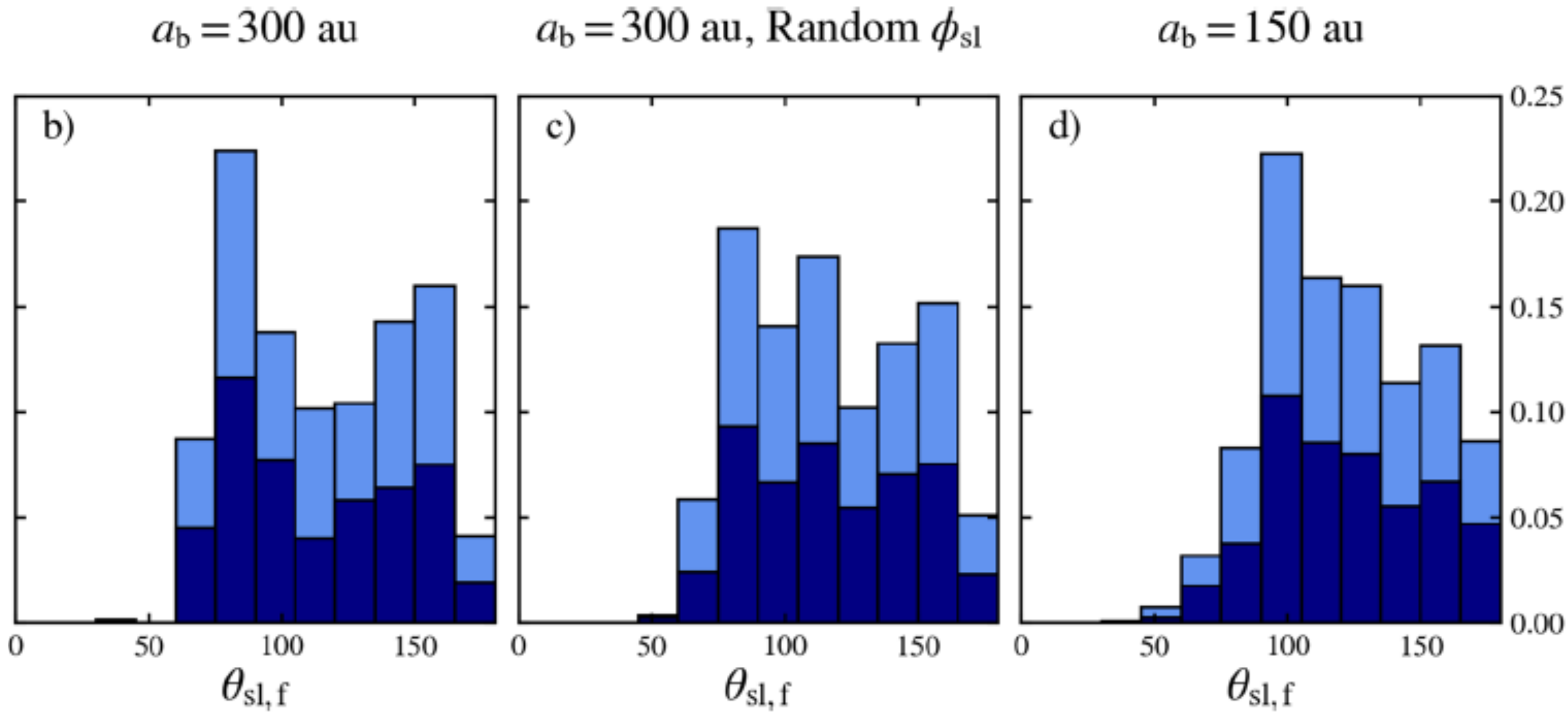

➔
Produce misaligned stellar spin with respect to disk axis (orbital axis of CJ)

Su & Lai 2024




After disk dissipates,
LK migration begins…

Su & Lai 2024




Vick, Su & Lai (2023)

# Summary I:
# Hot Jupiter Formation & (Stellar) Spin-Orbit Misalignment

## Formation/migration in protoplanetary disks

– Certainly apply to some HJs

-- Difficult to produce large misalignments

## High-Eccentricity Migration

-- Can naturally account for large stellar obliquities (spin-orbit dynamics important.)

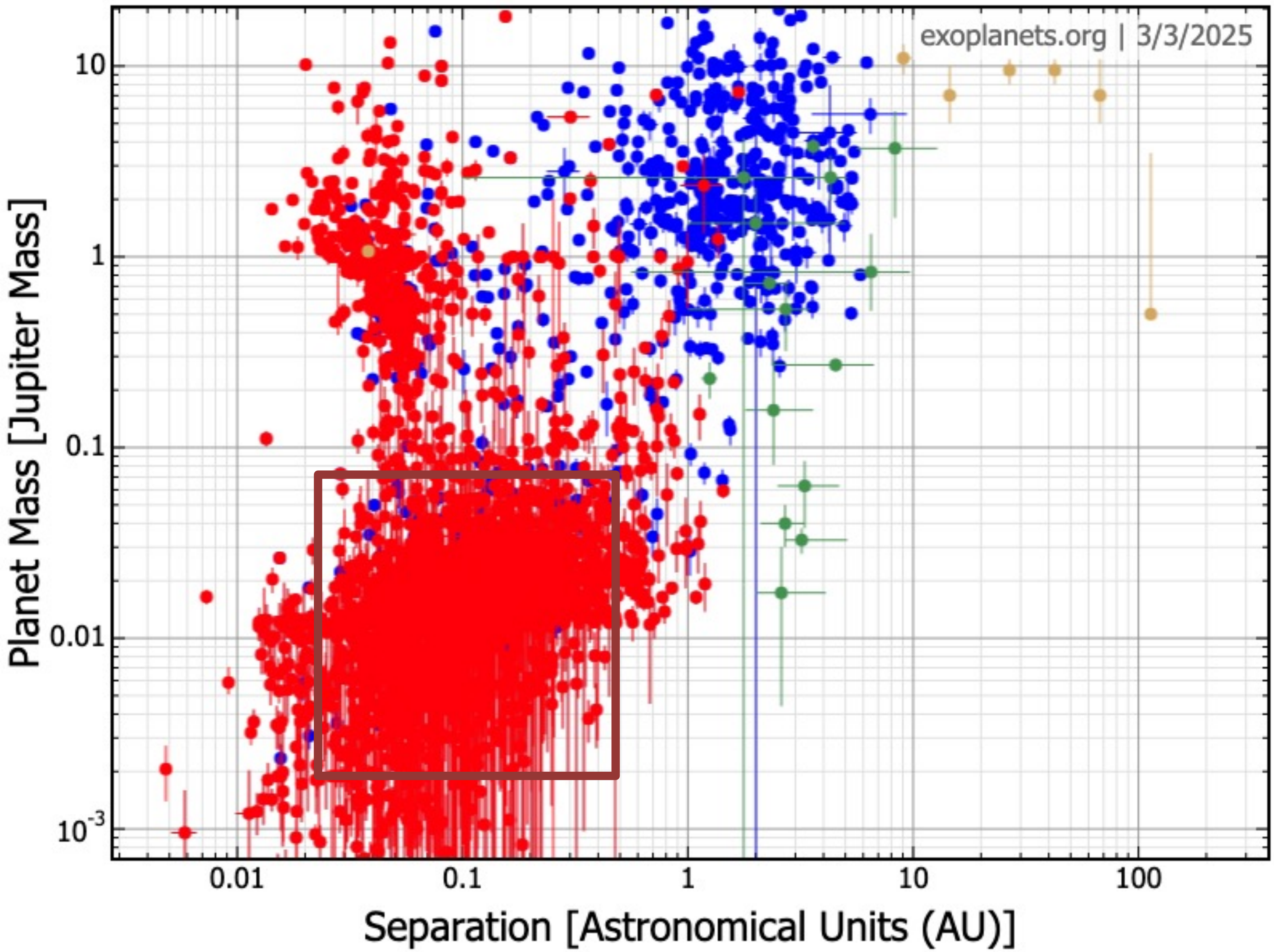
-- Combination of binary-disk-star interaction with LK migration generate broad obliquities (in agreement with data?)

Puzzles/Issues: e.g.

-- Does planet survive high-e migration (chaotic tides)?

-- Observed obliquities depend on mass/temperature of host stars…

# Super-Earths

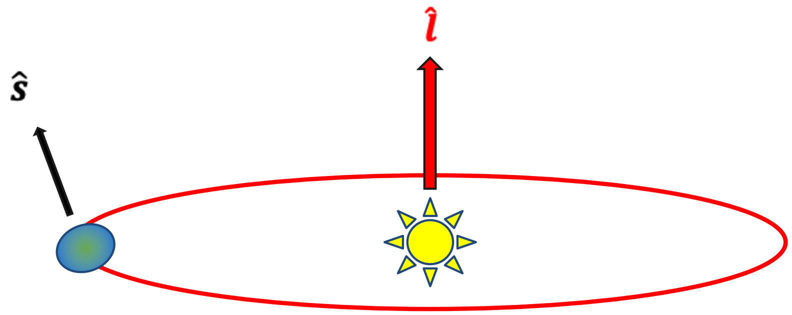


# Formation of Close-in Super-Earths

Migration (large vs small-scale),   in-Situ Formation

Characterizations: atmospheres…

# What are the (expected) spin obliquities of Super-Earths?




## Take-home message:

Some super-Earths in multi-planet systems have significant obliquities (despite strong tidal alignment torque)

# Planetary Obliquities

| | |
|---|---|
| Mercury | 0.03 |
| Venus | 177.36 |
| Earth | 23.44 |
| Mars | 25.19 |
| Jupiter | 3.13 |
| Saturn | 26.73 |
| Uranus | 97.77 |
| Neptune | 28.32 |

Affect the surface conditions, climate

-- Insolation
-- Atmosphere circulation
-- Milankovich cycles

Reflect the formation/evolution history of the planet

-- Giant impacts/collisions?
-- Secular spin-orbit resonances/overlaps (to be discussed)

# Obliquities of Exoplanets

Constraints for distant planetary-mass companions
(Bryan et al.2020,2021)

Future constraints for transiting planets?

Large obliquities can affect atmosphere conditions, transit signatures

Dynamical effects of finite obliquities:  Tidal dissipation/heating, orbital evolution

# Super-Earths:  How to generate obliquities?

# Planet Collisions

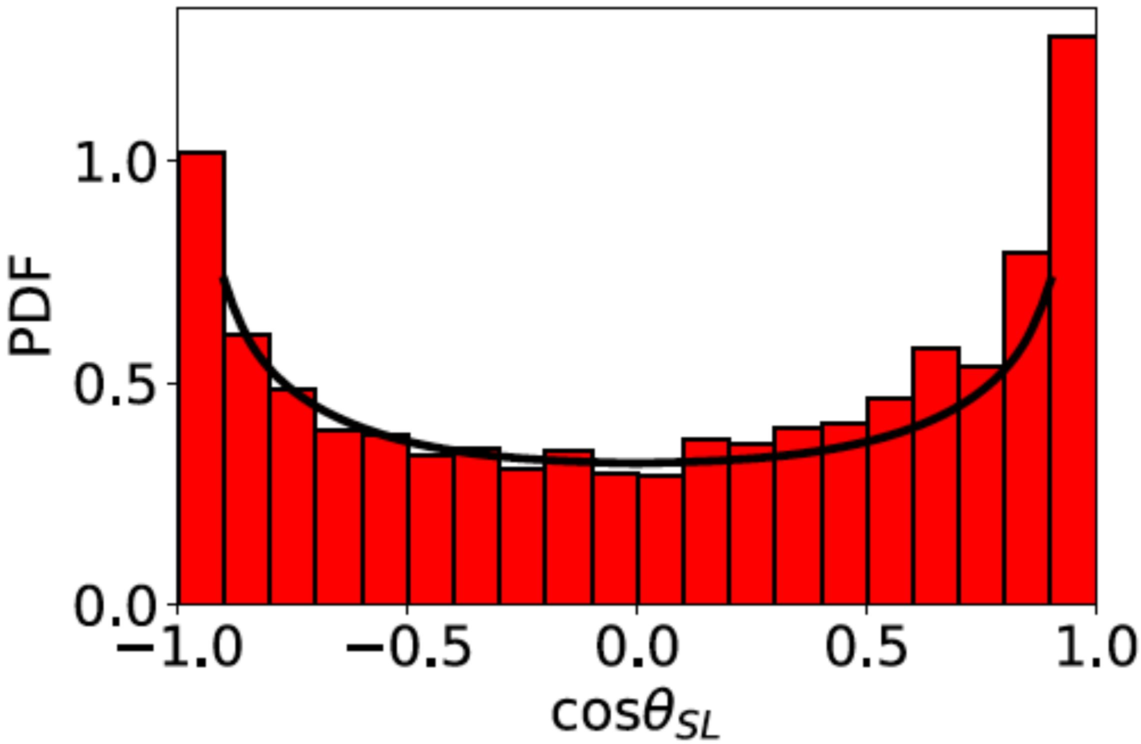
-- The formation of super-Earths likely involved giant impacts

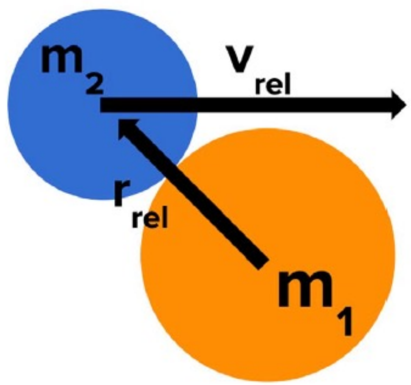
-- Many Kepler multi-planet systems are at the edge of dynamical instability (Volk & Gladman 2015; Pu & Wu 2015),

--> They likely have gone through a dynamical instability phase, where planet-planet collisions/mergers occurred

# Planetary Obliquities from Mergers

As long as initial mutual inclination >> R/a
==> Broad distribution of obliquities




Jiaru Li & Lai 2020
D.Wang, Li & Lai 2025

➔ Some super-Earths likely had large primordial obliquities

# Tidal dissipation in planet

tends to synchronize/align the spin with orbit

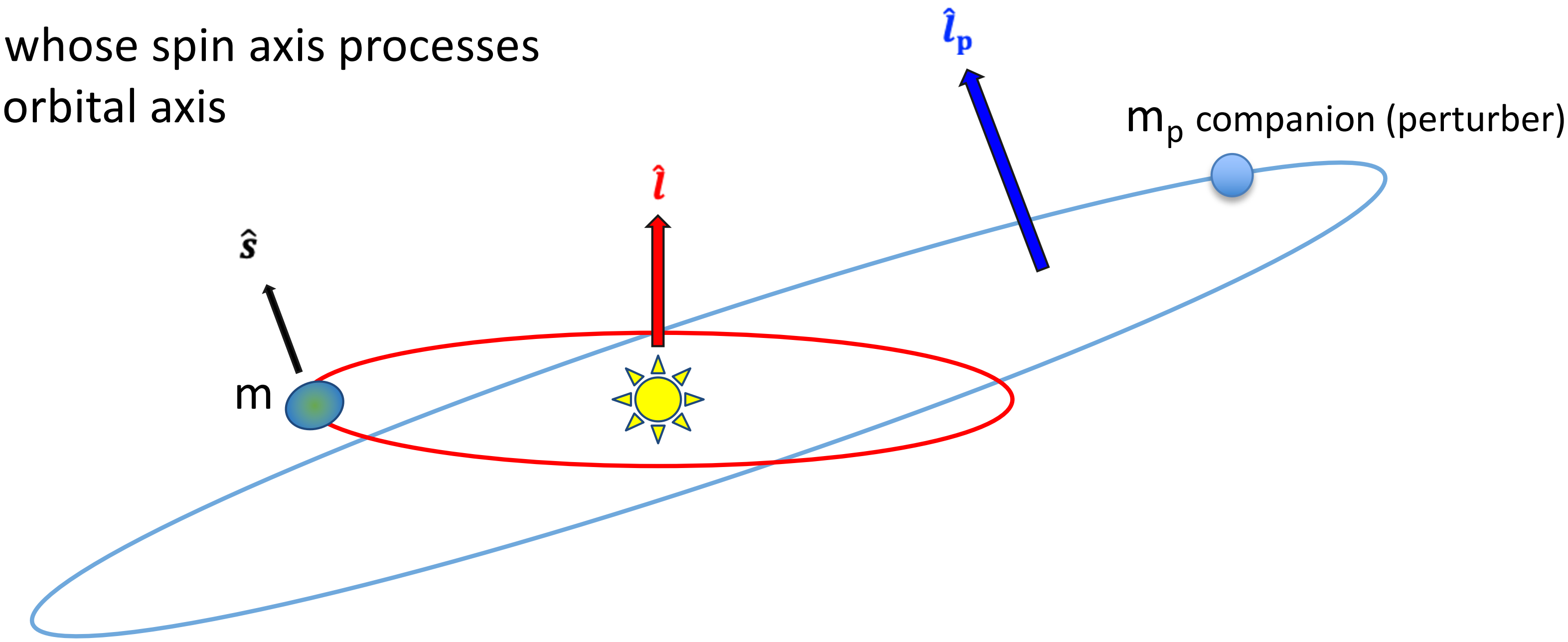
$$t_s \simeq 30 \left(\frac{Q}{10^3}\right) \left(\frac{M_\star}{M_\odot}\right)^{-3/2} \left(\frac{m}{4M_\oplus}\right) \left(\frac{R}{2R_\oplus}\right)^{-3} \left(\frac{a}{0.4\,\mathrm{au}}\right)^{9/2} \mathrm{Myr}$$

➔ Isolated super-Earths will have aligned spin

What about super-Earth with a companion (perturber)?

# Dynamics of Colombo's Top

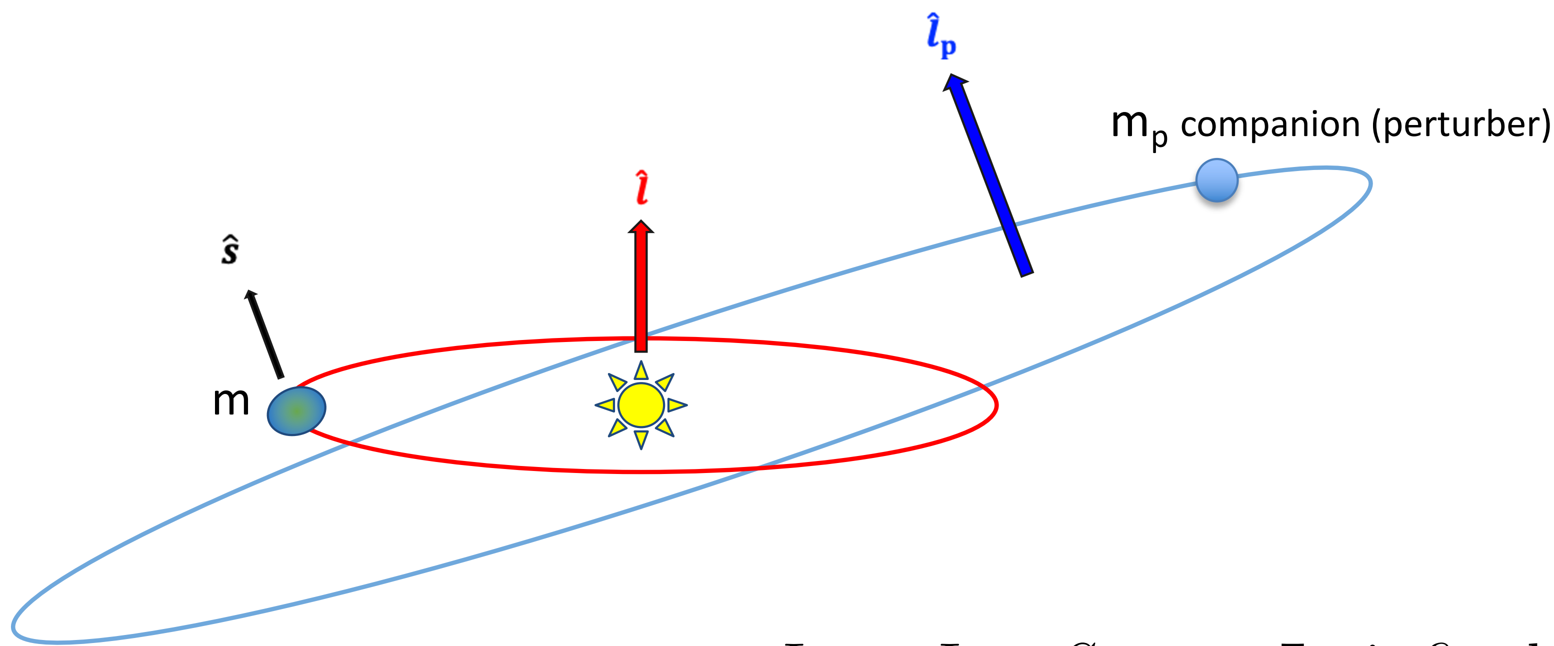
A rotating planet whose spin axis processes around a varying orbital axis




Colombo 66; Peale 69; Ward 75; Henrard & Murigande 87.....

Su & Lai 2021, 2022a,b

# Dynamics of Colombo's Top




$$L_p \gg L \gg S \quad \Longrightarrow \mathbf{L}_p \text{ is fixed}$$

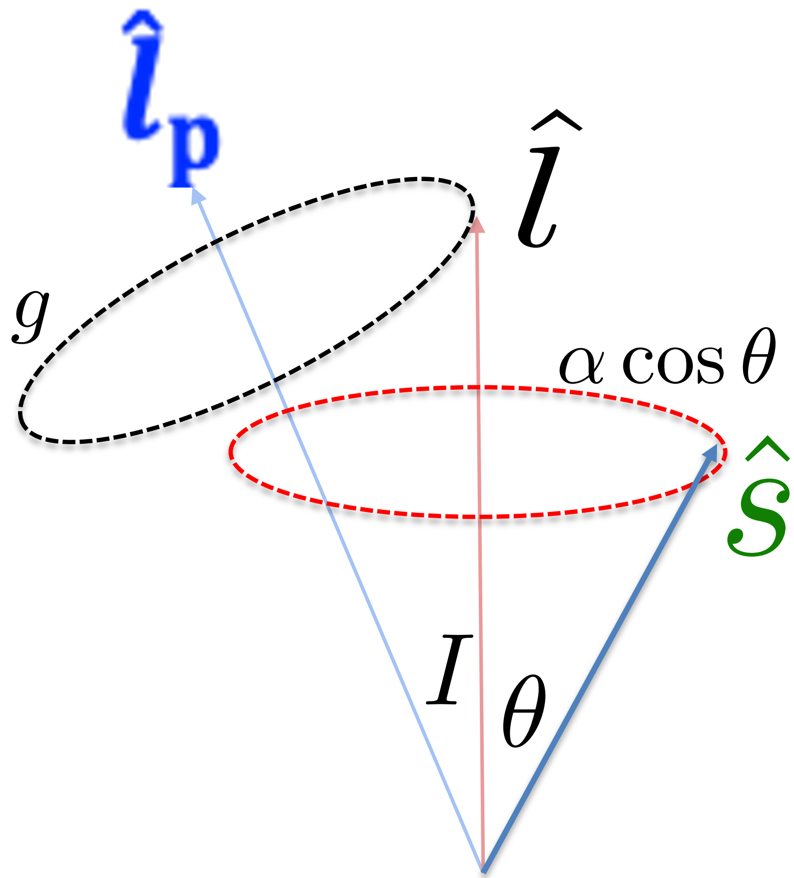
$$\frac{d\hat{\mathbf{l}}}{dt} = \omega_{\mathrm{lp}} \left( \hat{\mathbf{l}} \cdot \hat{\mathbf{l}}_{\mathrm{p}} \right) \left( \hat{\mathbf{l}} \times \hat{\mathbf{l}}_{\mathrm{p}} \right) \equiv -g \left( \hat{\mathbf{l}} \times \hat{\mathbf{l}}_{\mathrm{p}} \right), \qquad \text{(Orbit precession)}$$

$$\omega_{\mathrm{lp}} \equiv -\frac{g}{\cos I} = \frac{3m_{\mathrm{p}}}{4M_\star} \left( \frac{a}{a_{\mathrm{p}}} \right)^3 n$$

$$\frac{d\hat{\mathbf{s}}}{dt} = \omega_{\mathrm{sl}} \left( \hat{\mathbf{s}} \cdot \hat{\mathbf{l}} \right) \left( \hat{\mathbf{s}} \times \hat{\mathbf{l}} \right) \equiv \alpha \left( \hat{\mathbf{s}} \cdot \hat{\mathbf{l}} \right) \left( \hat{\mathbf{s}} \times \hat{\mathbf{l}} \right), \qquad \text{(Spin precession)}$$

$$\omega_{\mathrm{sl}} \equiv \alpha = \frac{3GJ_2 m R^2 M_\star}{2a^3 I \Omega_{\mathrm{s}}} = \frac{3k_q}{2k} \frac{M_\star}{m} \left( \frac{R}{a} \right)^3 \Omega_{\mathrm{s}}$$

In general, the obliquity varies in time…

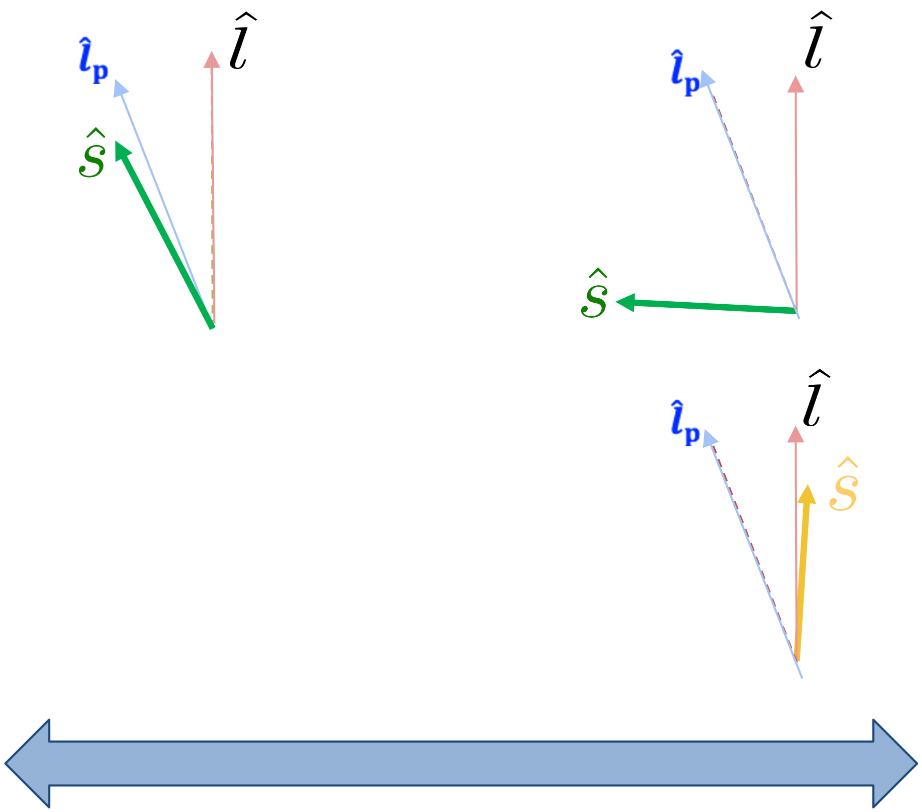


In general, the obliquity varies in time… Is there equilibrium?

# Equilibria: Cassini States

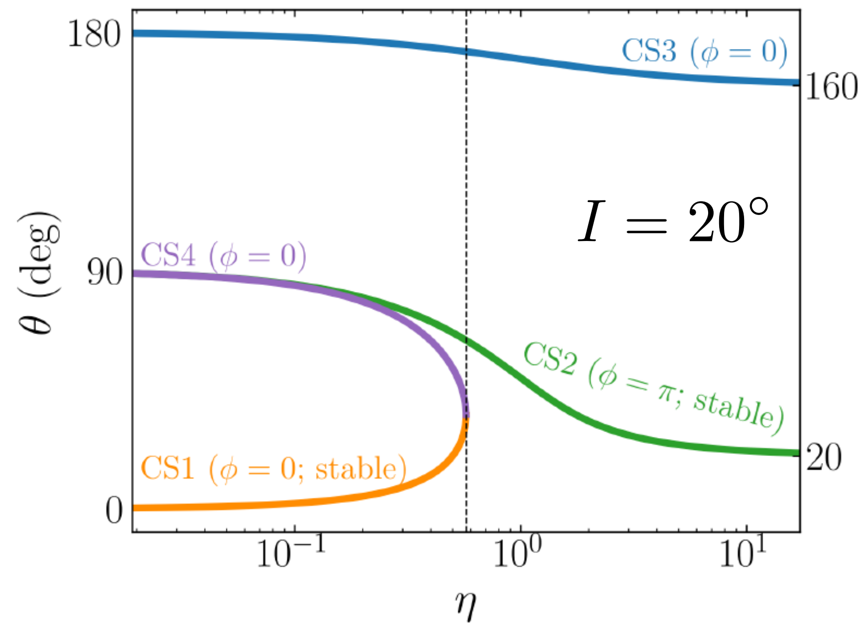
($\theta$ =const, or $\hat{s}$ =const in rotating frame)

$$\left(\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t}\right)_{\mathrm{rot}} = \alpha\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)\left(\hat{\mathbf{s}}\times\hat{\mathbf{l}}\right) + g\left(\hat{\mathbf{s}}\times\hat{\mathbf{l}}_{\mathrm{p}}\right) = 0$$



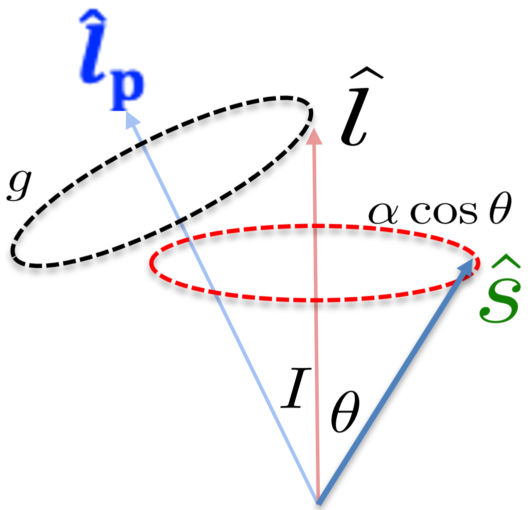

$\eta \gg 1$
(fast orbital precession)

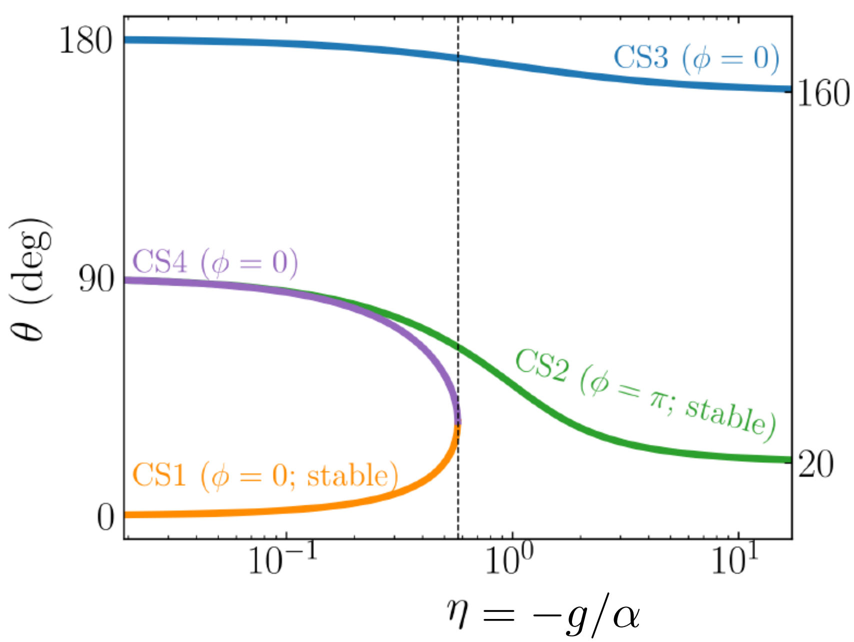
$\eta \ll 1$
(fast spin precession)

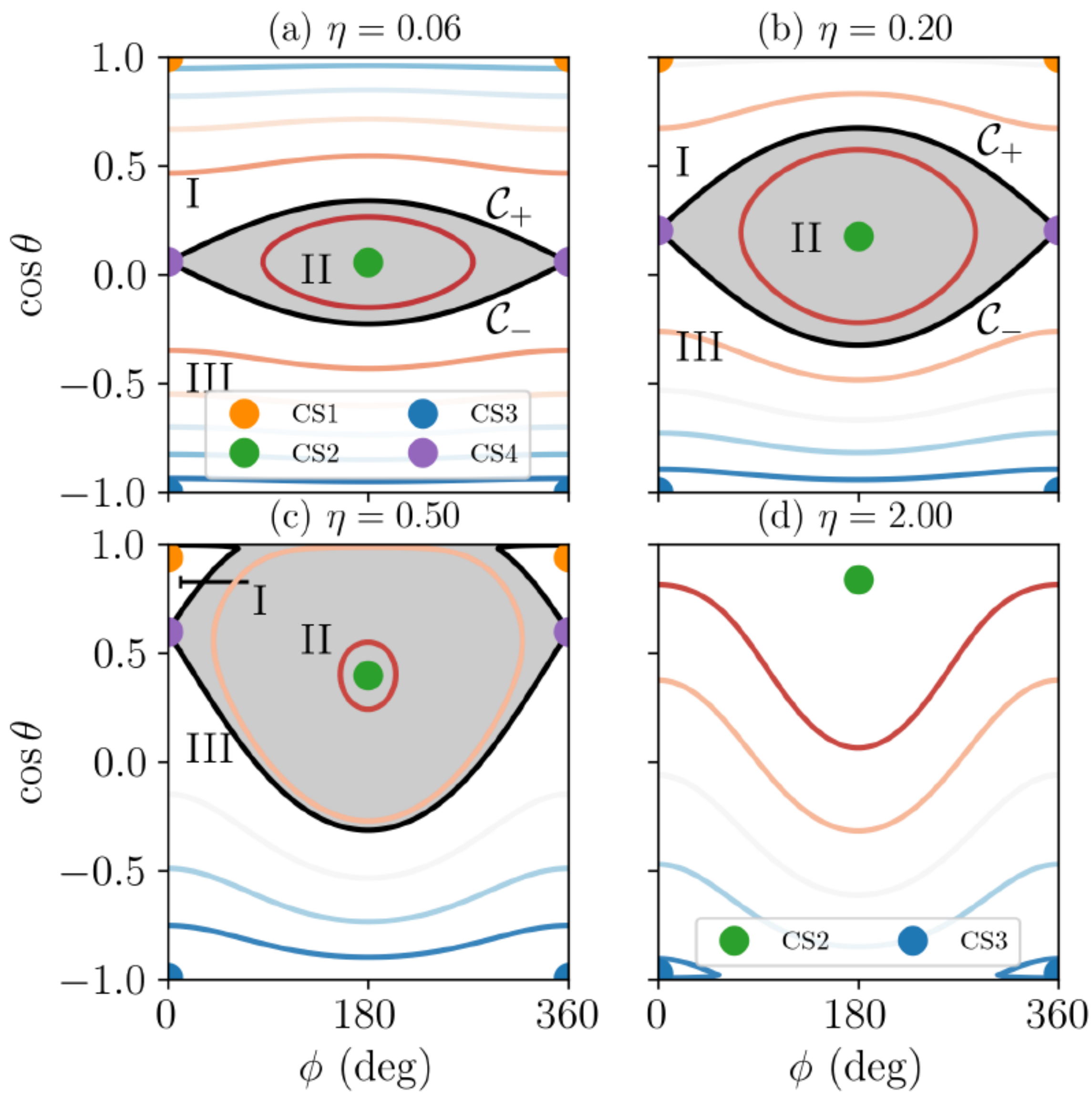



$$\eta \equiv -\frac{g}{\alpha}$$

# Phase portrait and Spin-Orbit Resonance

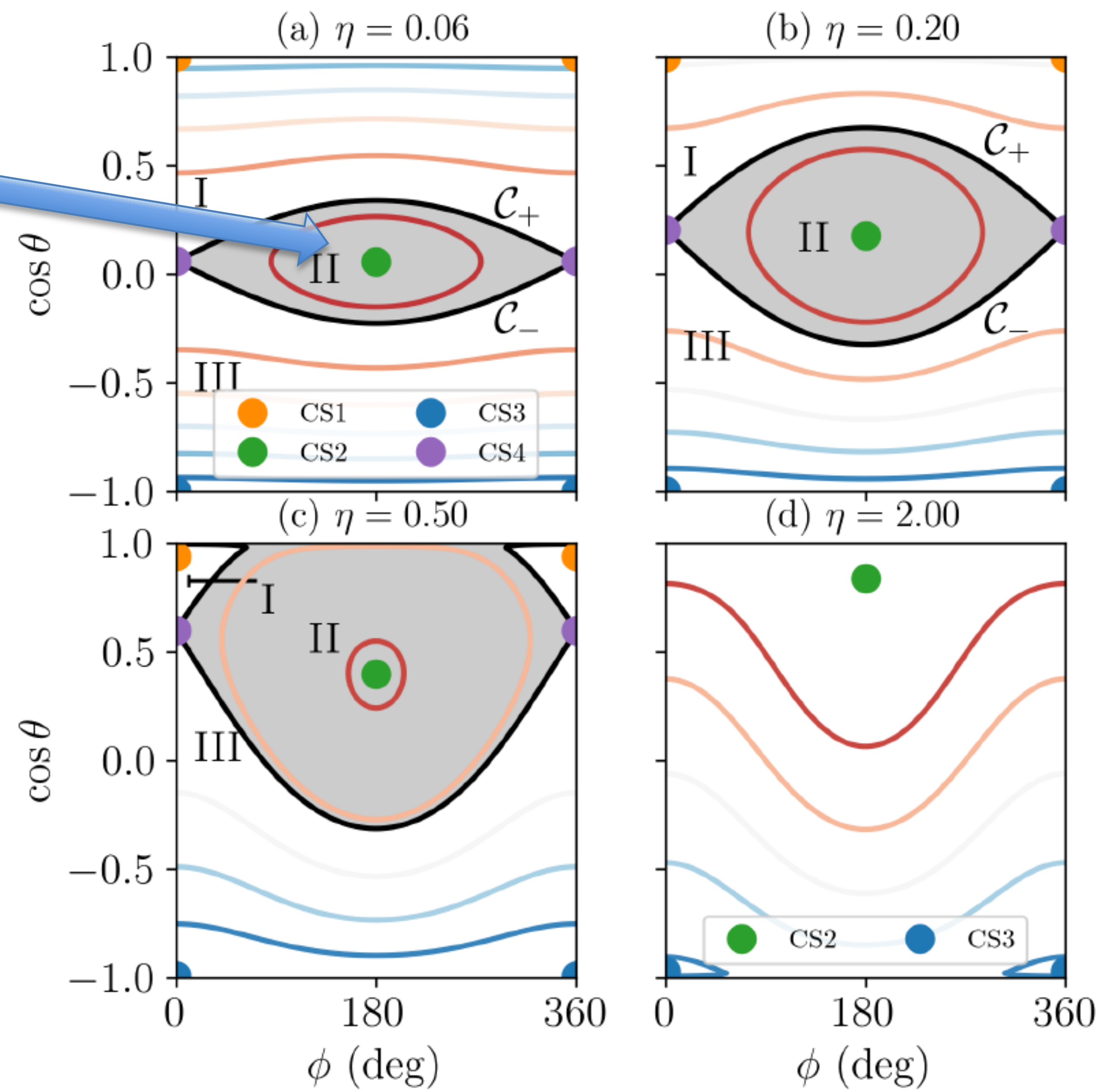


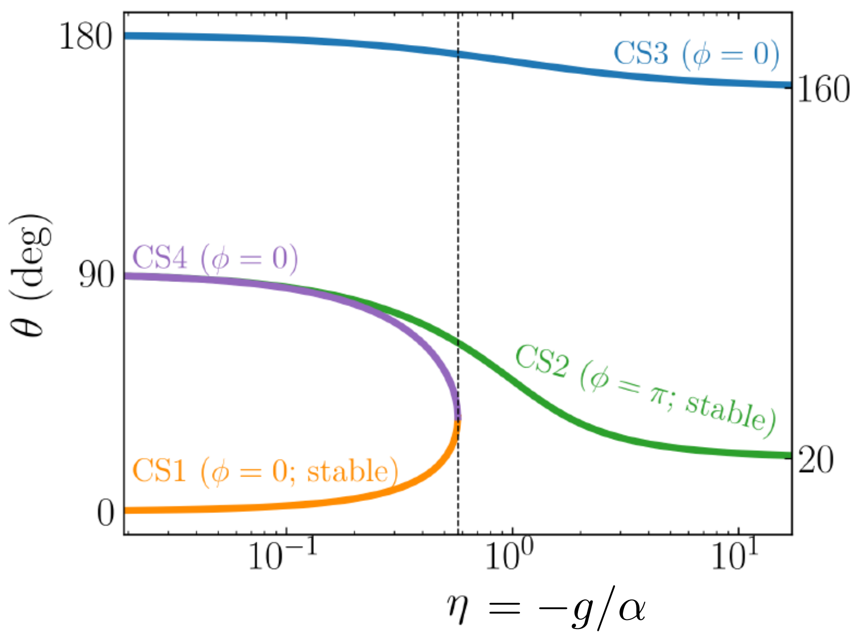



# Phase portrait and Spin-Orbit Resonance

Resonance
$\alpha \cos\theta \sim |g|$

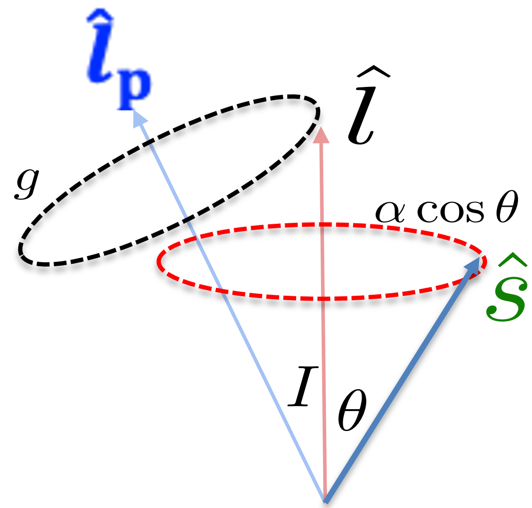


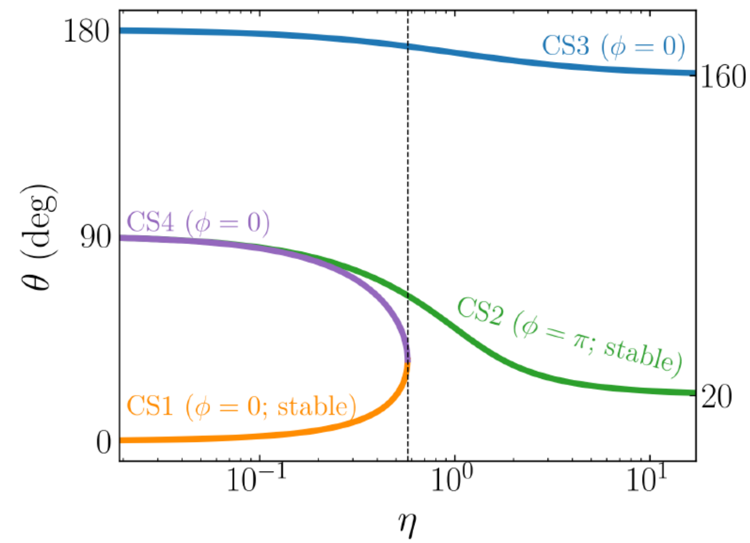



# Add tidal torque...

Tidal torque tries to push $\hat{s}$ toward $\hat{l}$, but $\hat{l}$ is changing...

$\hat{S}$ evolves toward one of the equilibria (Cassini states)




$$\omega_{\rm sl} \equiv \alpha = \frac{3GJ_2 mR^2 M_\star}{2a^3 I\Omega_{\rm s}} = \frac{3k_q}{2k}\frac{M_\star}{m}\left(\frac{R}{a}\right)^3 \Omega_{\rm s}$$

Spin precession

$$\omega_{\rm lp} \equiv -\frac{g}{\cos I} = \frac{3m_{\rm p}}{4M_\star}\left(\frac{a}{a_{\rm p}}\right)^3 n.$$

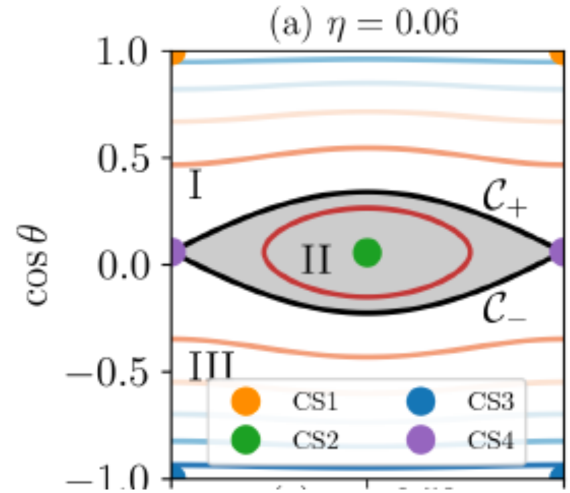
Orbital precession

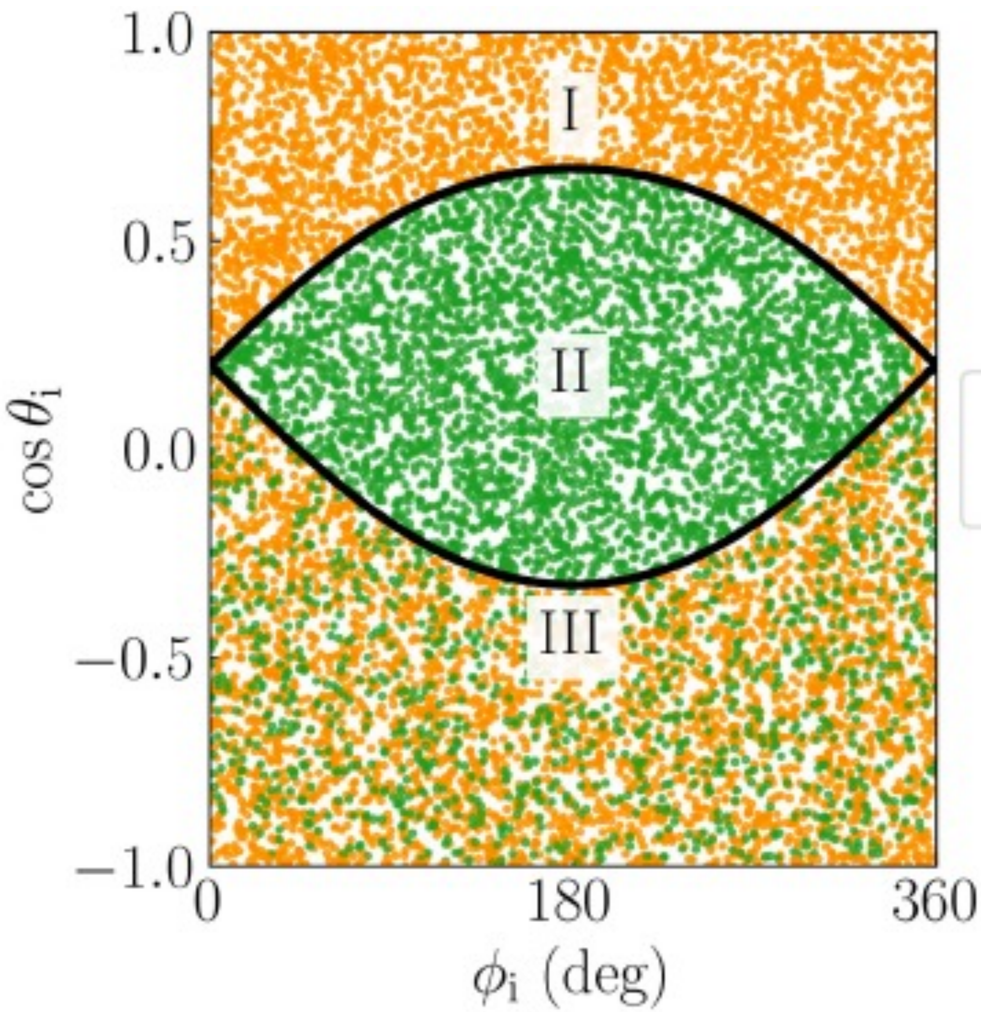
$$\eta \equiv -\frac{g}{\alpha} \sim 0.06\left(\frac{a}{0.04\,\mathrm{au}}\right)^3\left(\frac{m_p}{10M_\oplus}\right)\left(\frac{1.3a}{a_p}\right)^3$$

## Final Outcomes with tidal alignment torque

For $\eta \lesssim 1$ (fast spin precession, weak perturber):

$\hat{S}$ evolves towards CS1 or CS2



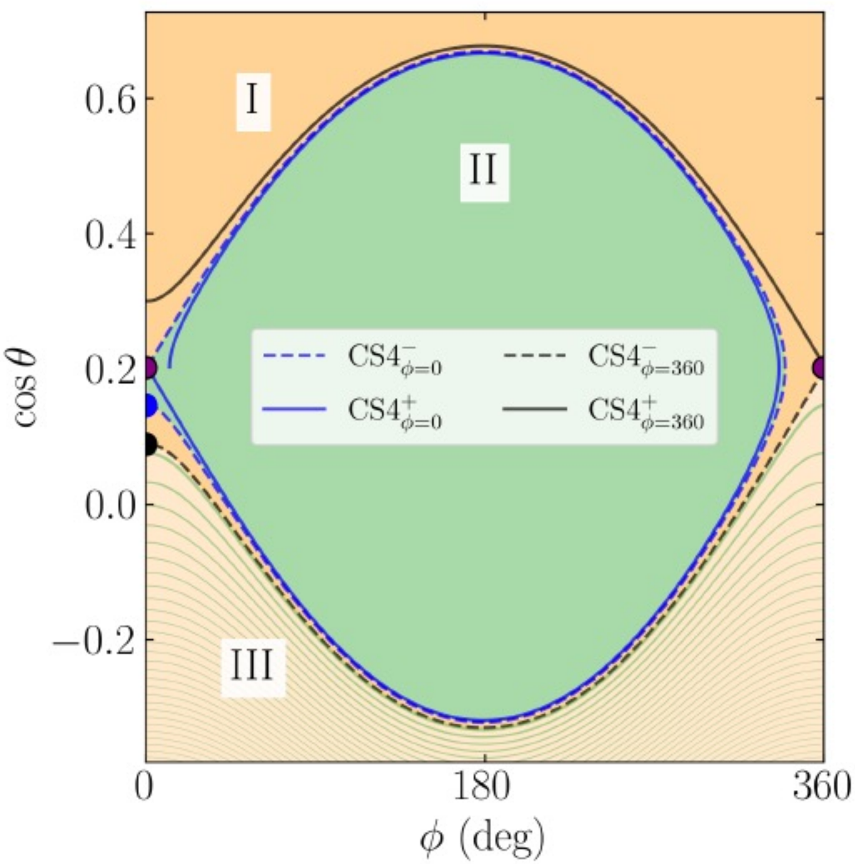



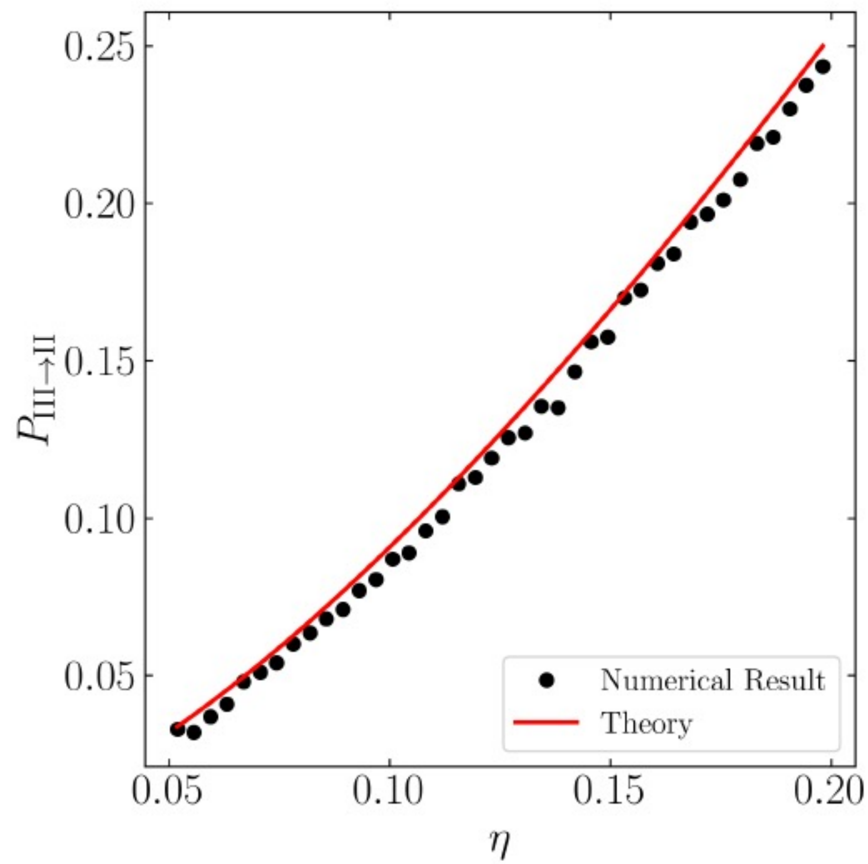

Zone I → CS1 (low obliquity)

Zone II → CS2 (high obliquity)

Zone III → Either CS1 or CS2, probabilistic

Separatrix Crossing:   Transition Probability

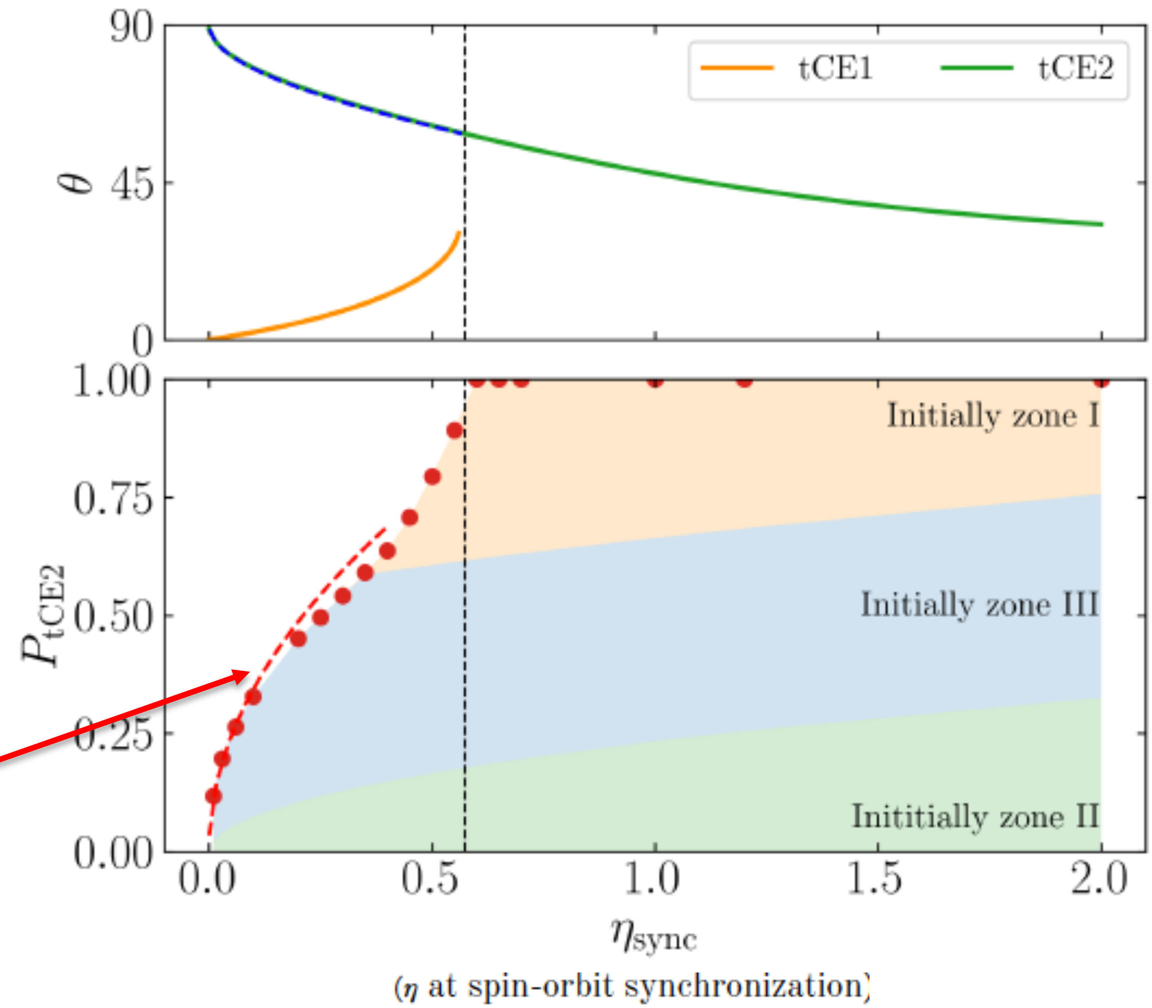


# Recap:

In the presence of companion, tidal torque does not erase obliquity;
Instead, it drives the spin axis towards a "tidal-Cassini" equilibrium, tCS1 or tCS2, which can have appreciable obliquity

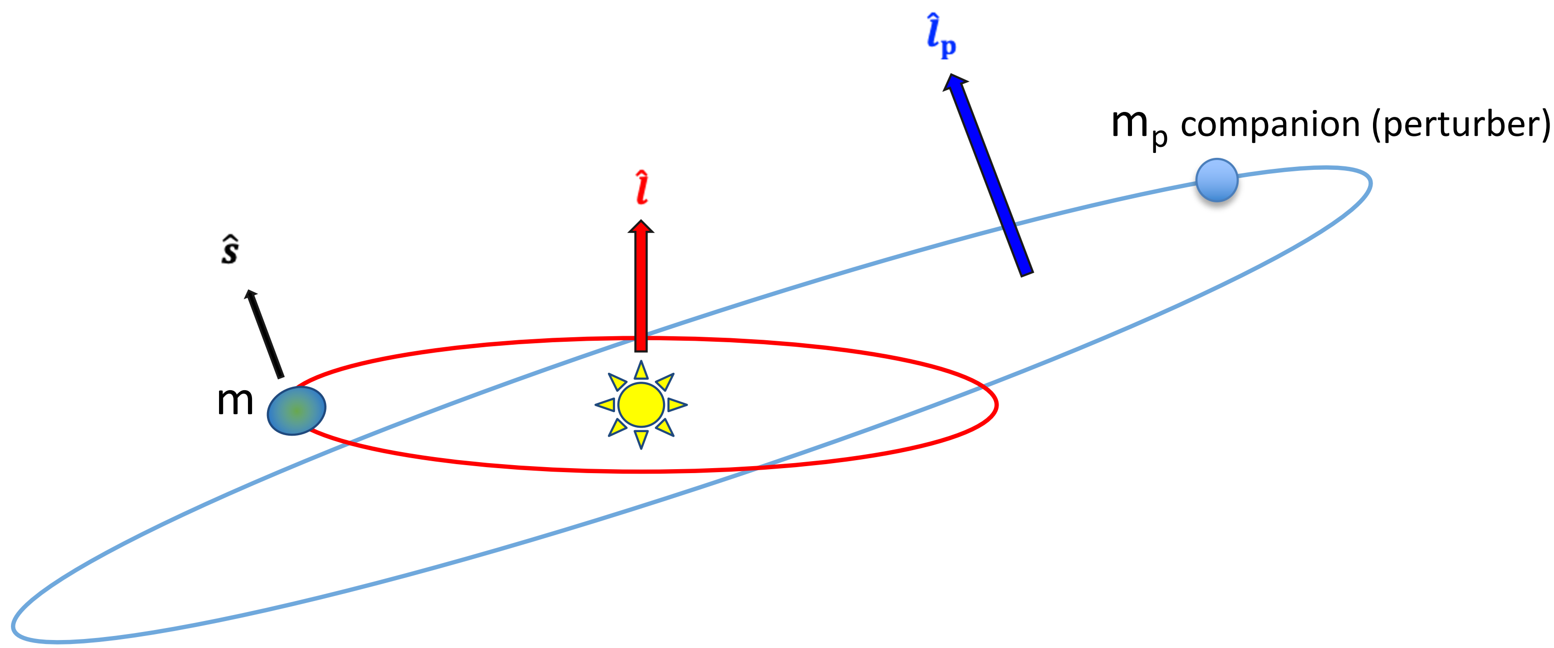
Starting from isotropic spin orientations:



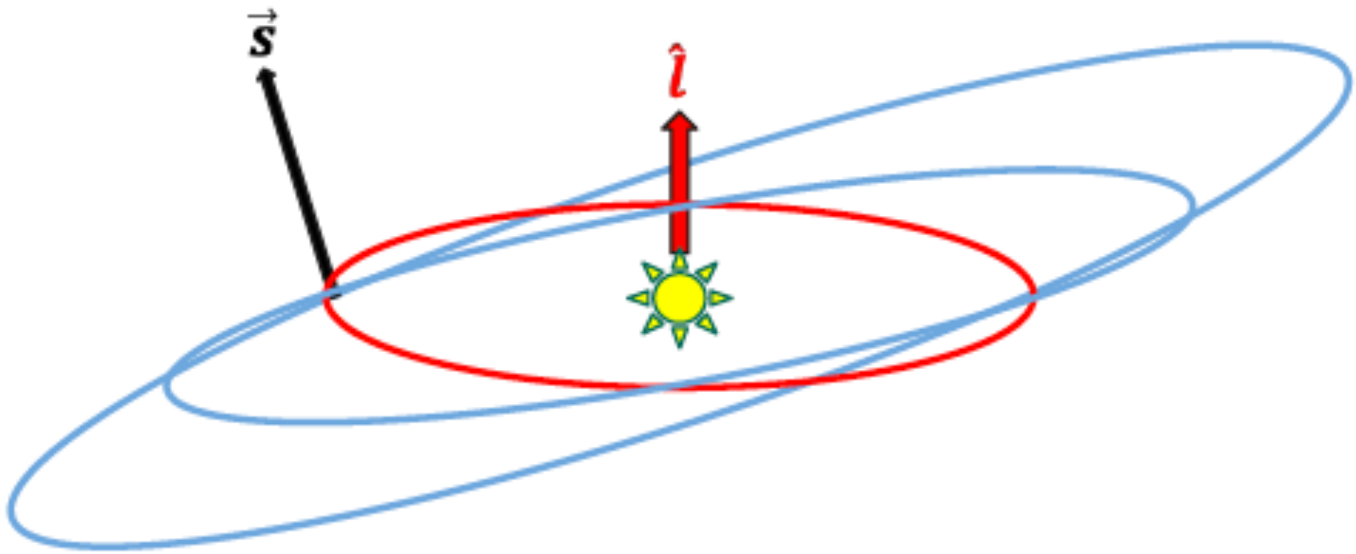

Probability of evolving
Into tCE2

$$P_{\mathrm{tCE2}} \simeq \frac{4\sqrt{\eta_{\mathrm{sync}}}\sin I}{\pi}\left[\sqrt{n/\Omega_{\mathrm{s,i}}} + \frac{3}{2\left(1+\sqrt{n/\Omega_{\mathrm{s,i}}}\right)}\right]$$

# So far…



## What happens if the super-Earth has 2 (or more) companions?

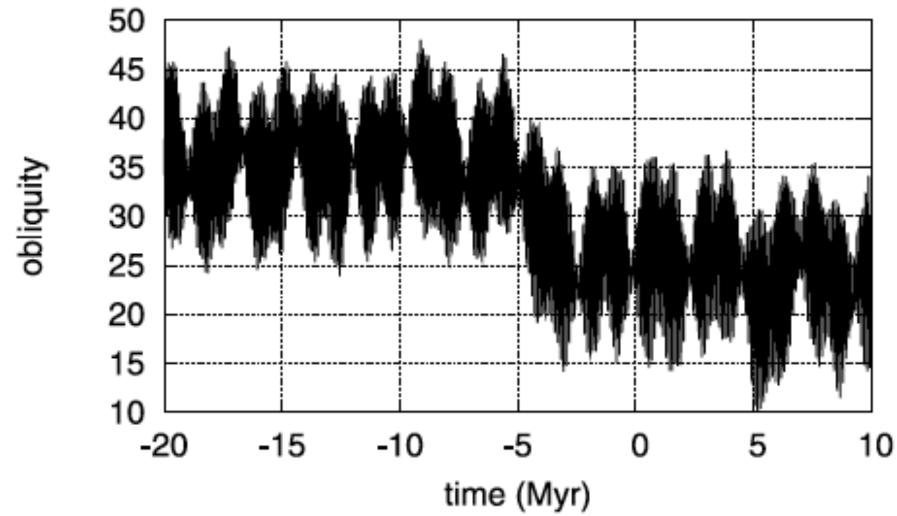



$$\frac{\mathrm{d}\hat{\mathbf{s}}}{\mathrm{d}t} = \alpha\left(\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}\right)\left(\hat{\mathbf{s}}\times\hat{\mathbf{l}}\right)$$

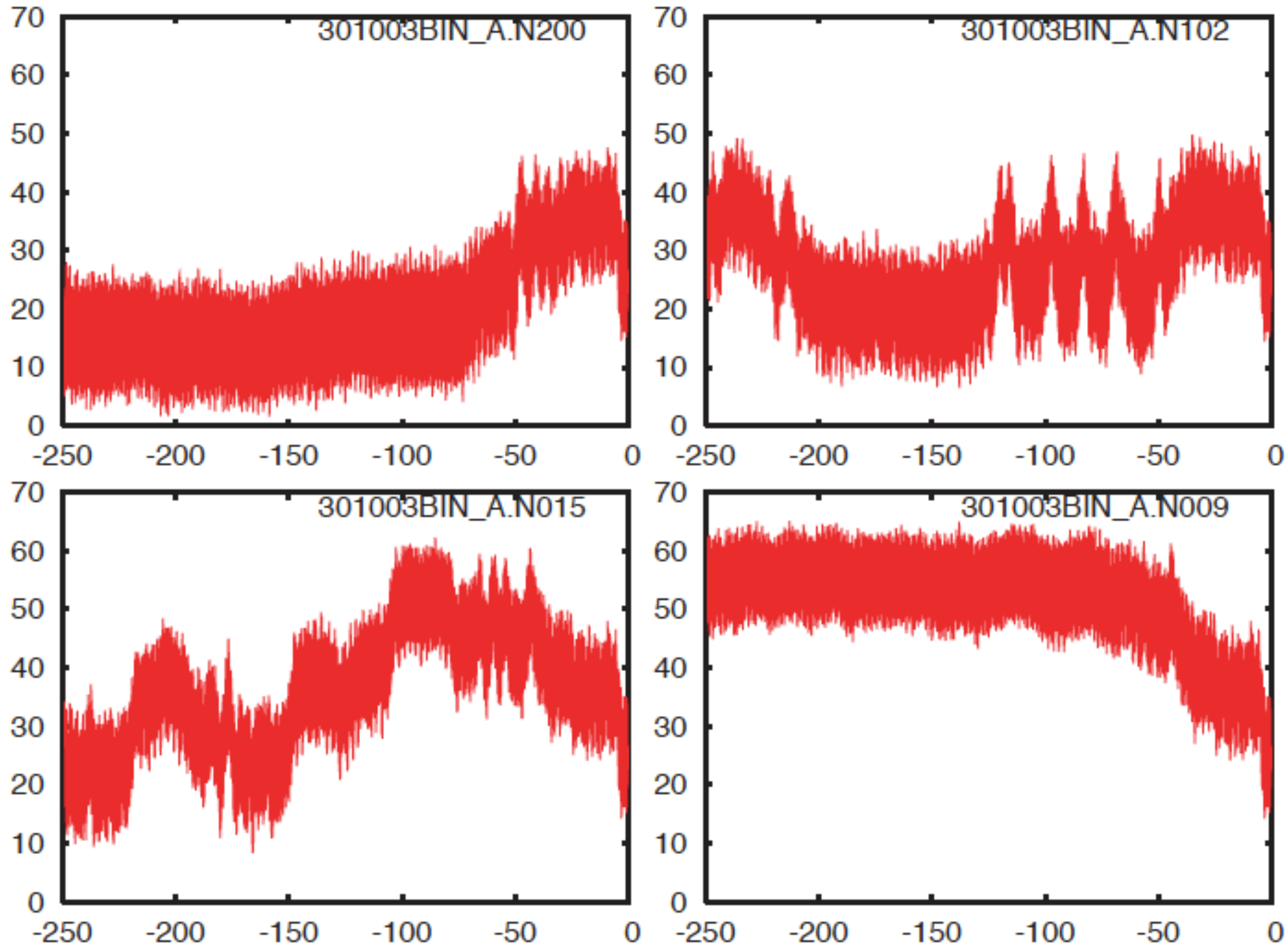
$\hat{\mathbf{l}}$ precesses/wobbles with 2 (or more) frequencies

# Mars' obliquity evolution

Chaotic, due to overlapping resonances

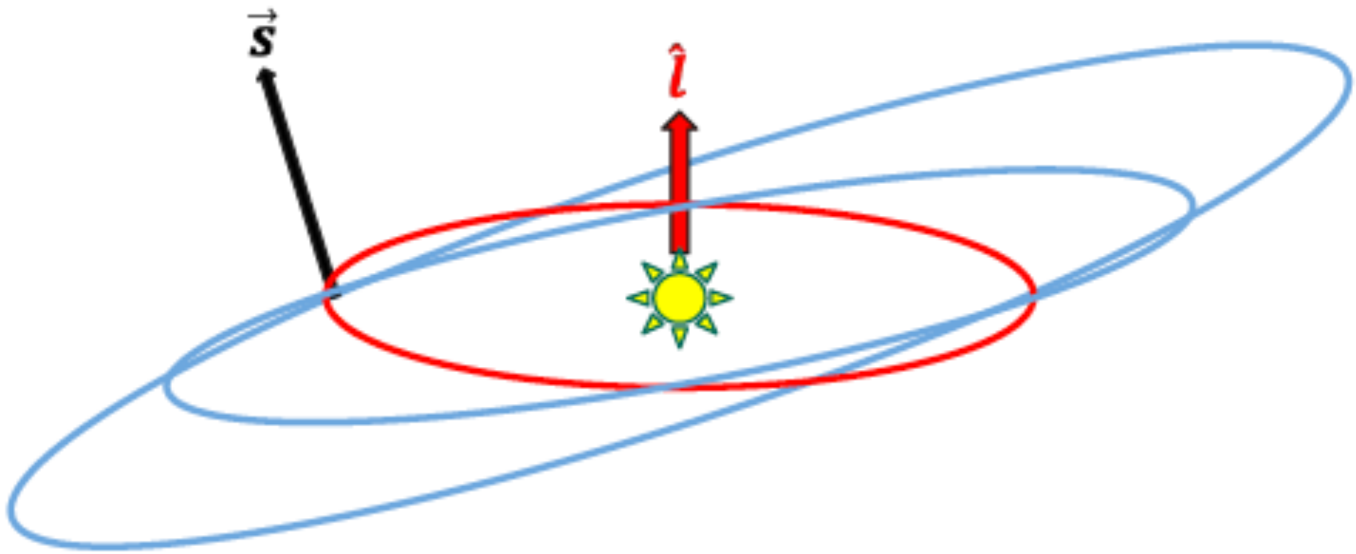



Possible realizations of Mars' obliquity evolution over the last 200 Myrs.




Laskar et al. 2004

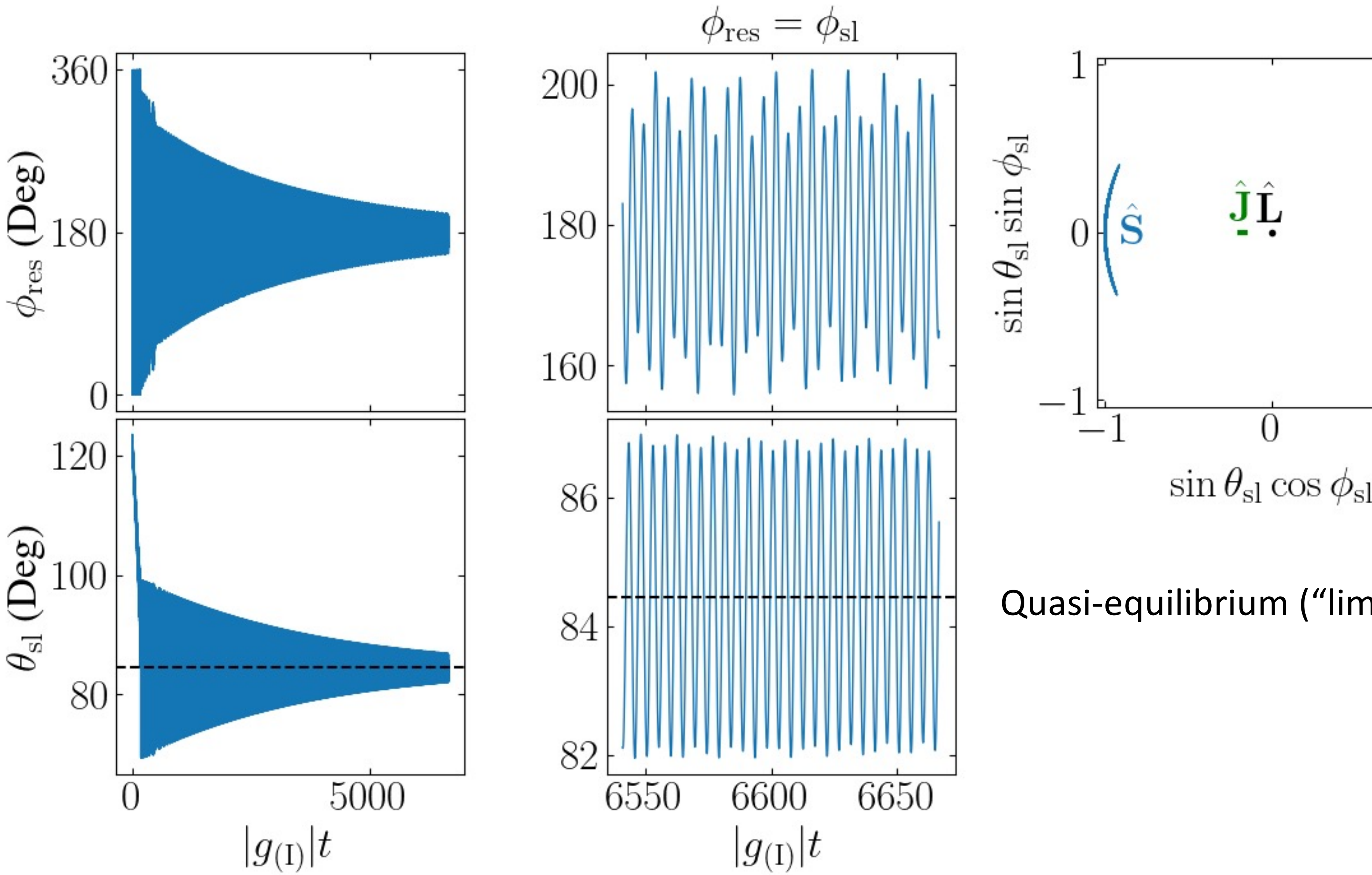
# Super-Earth with 2 companions, including tidal alignment torque




$$\frac{d\hat{\mathbf{s}}}{dt} = \alpha \left(\hat{\mathbf{s}} \cdot \hat{\mathbf{l}}\right) \left(\hat{\mathbf{s}} \times \hat{\mathbf{l}}\right) + \text{tidal torque}$$

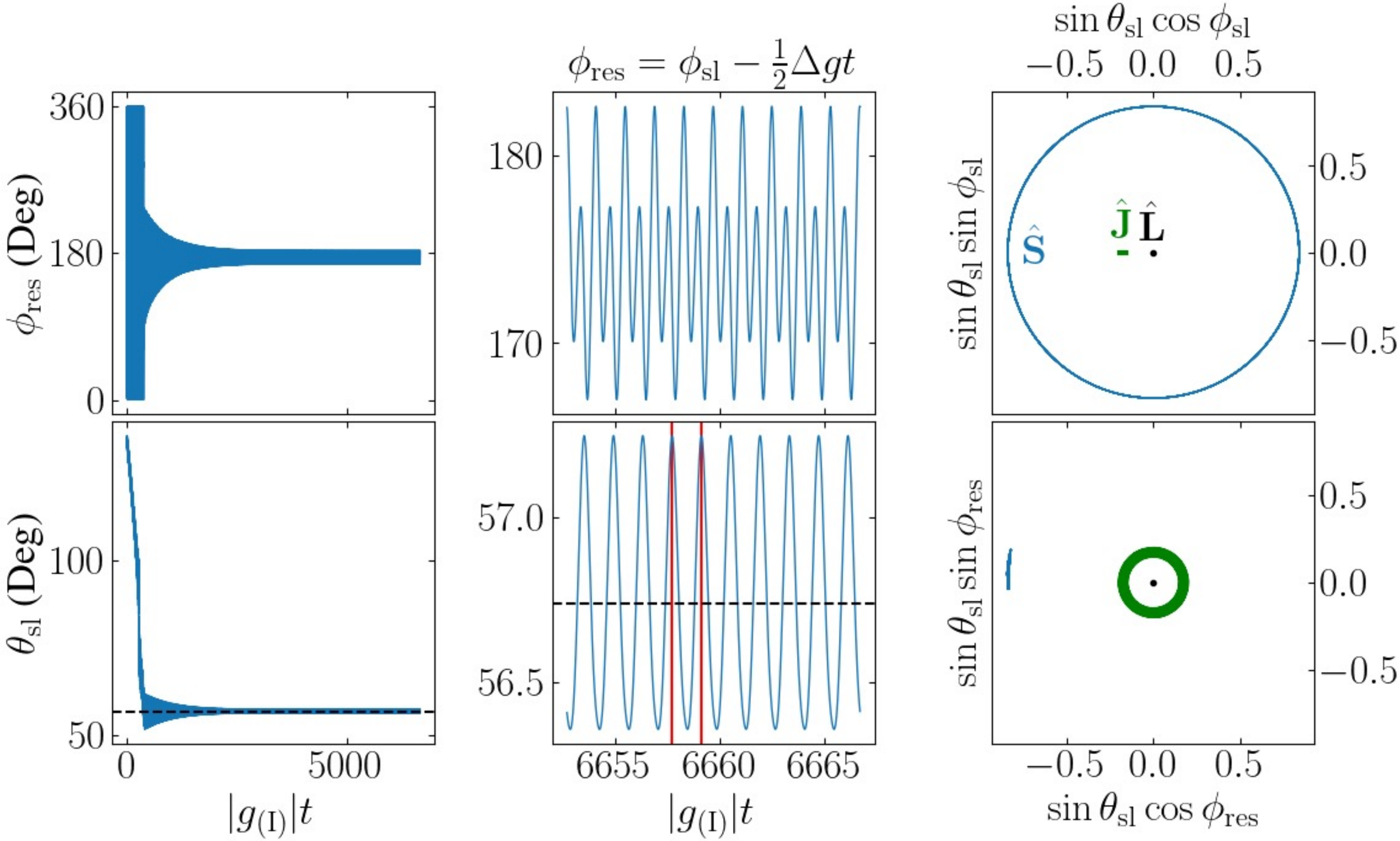
$\hat{\mathbf{l}}$ precesses/wobbles with 2 frequencies

# Spin axis evolution of a super-Earth with 2 companions, including tidal torque



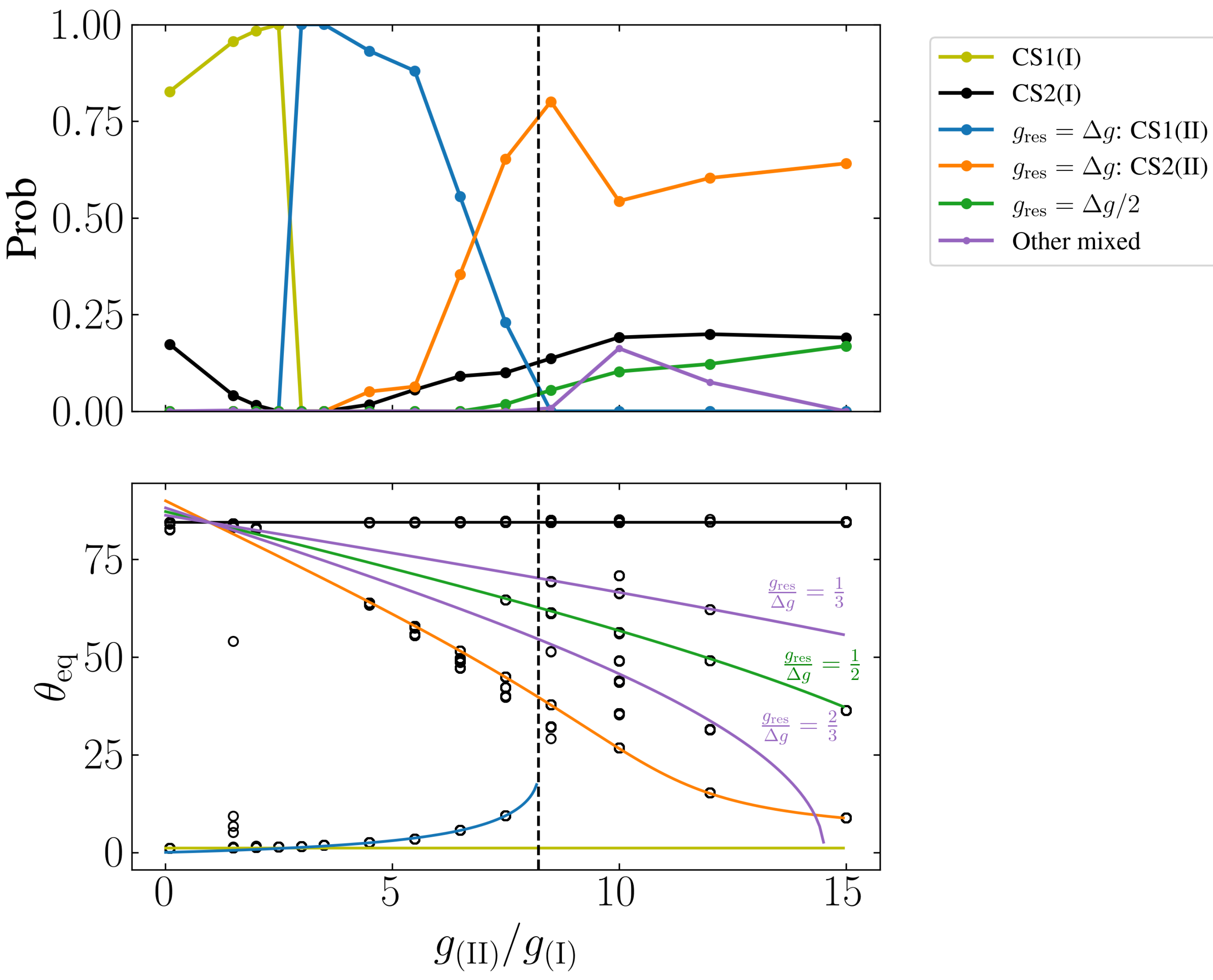

Quasi-equilibrium ("limit cycle")

# Spin axis evolution of a super-Earth with 2 companions, including tidal torque




Mixed-mode “equilibrium”

# Spin axis evolution of a super-Earth with 2 companions, including tidal torque



# Recap:

In the presence of companion(s), tidal torque does not erase obliquity;
Instead, it drives the spin axis towards one of the "tidal-Cassini equilibria",
maintaining a permanent obliquity

## Dynamical roles of super-Earth obliquities:

-- Tidal heating
-- Orbital decay
e.g. formation of ultra-short period planets (?)

See Millholland & Spalding 2020
Su & Lai 2022a (Paper II)

# Summary

## Hot Jupiters:

Stellar obliquities provide diagnostics for formation/migration mechanisms

Binary-disk-star interaction with high-e migration generate broad obliquities
(in agreement with data?)

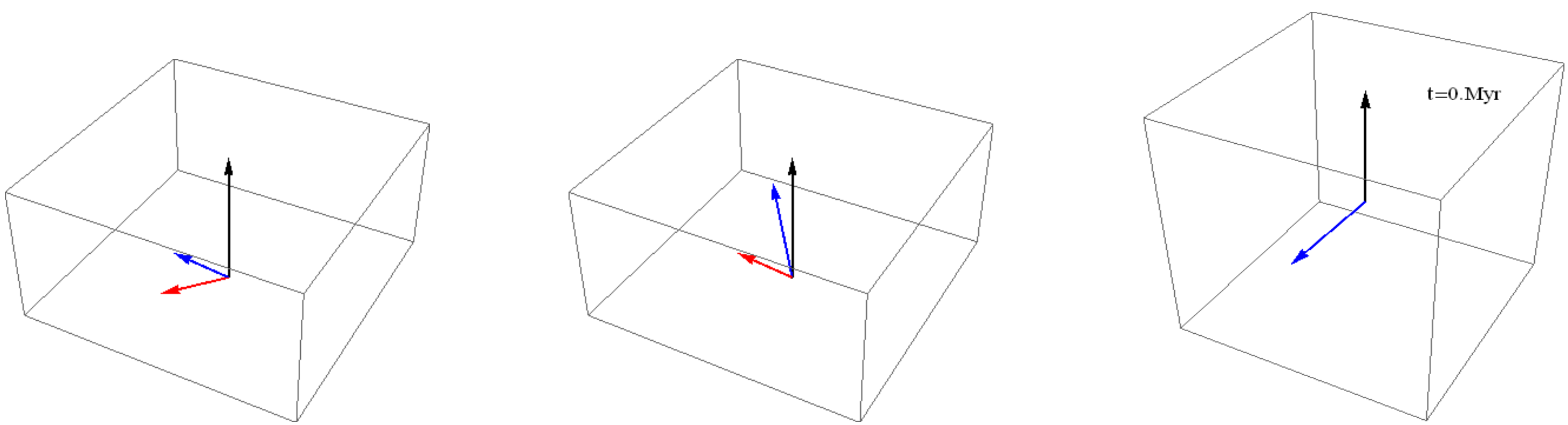
## Super-Earths:

Likely have experienced collisions

➔broad distribution of primordial planet obliquities

With tidal alignment torque, spin axis evolves towards one of the
“tidal-Cassini” equilibria, with non-trivial obliquity

➔ Some super-earths may/likely have appreciable obliquities



# Planet Collisions

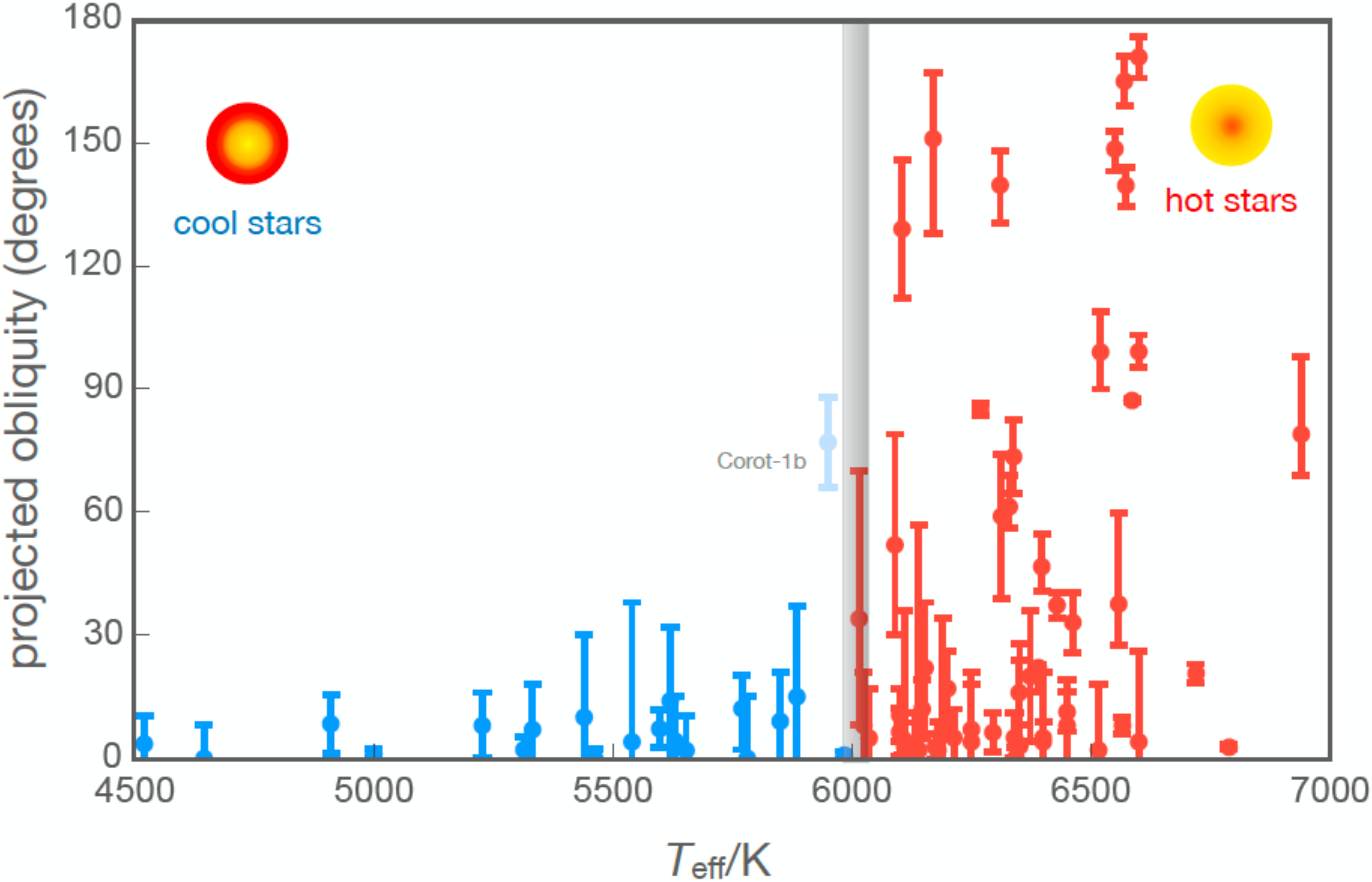
## Likely have occurred for super-Earths in multi-planet systems

-- Many Kepler multi-planet systems are at the edge of dynamical instability
(Volk & Gladman 2015; Pu & Wu 2015)

-- Super-Earths form in gas disks:
Migration and eccentricity damping → leading to densely-packed systems;
After the gas disk dissipates, mutual gravitational interaction causes instability;
➔ the system settles down to "metastable" state.

Outcomes of orbital instability of multiple super-Earths:

**planet-planet collisions/mergers**




Spalding & Winn 2022